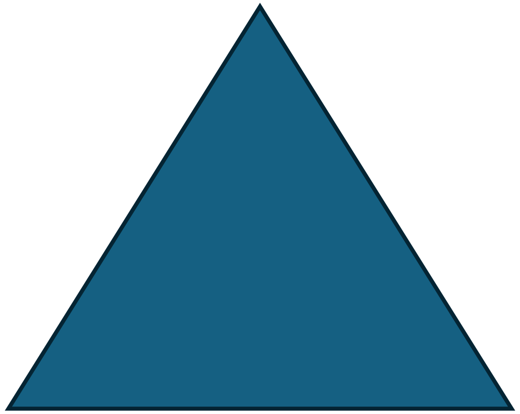
Triangles:



(A Tale of Obsession)

Dr. Mary Anne Clark
(AKA Max Chatnoir)

Triangles

- Triangles are 3-sides polygons: Trigons

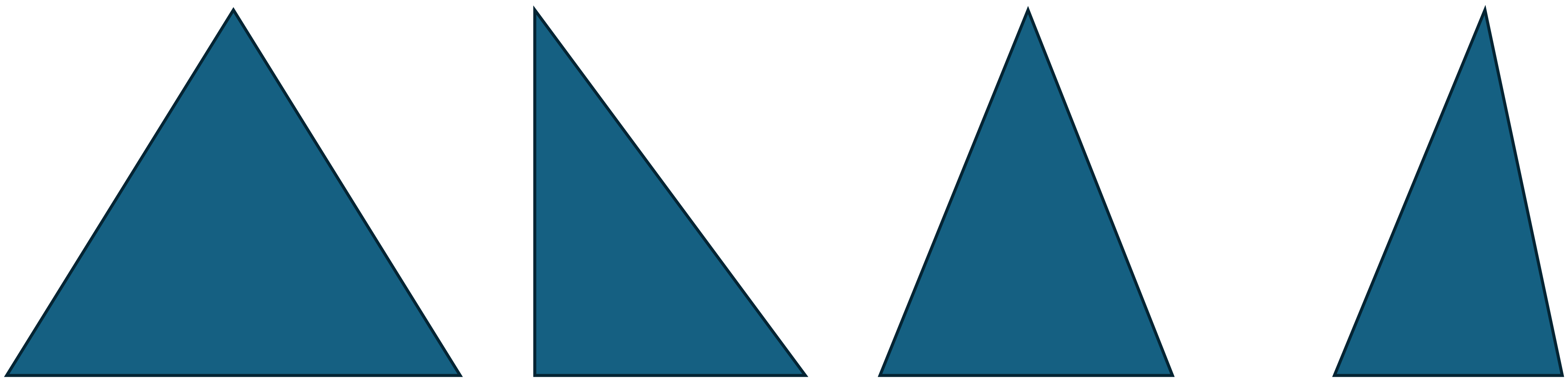
- You will recognize this as related to

TRIGONOMETRY

A realm of mathematics related to the measurement of triangles.

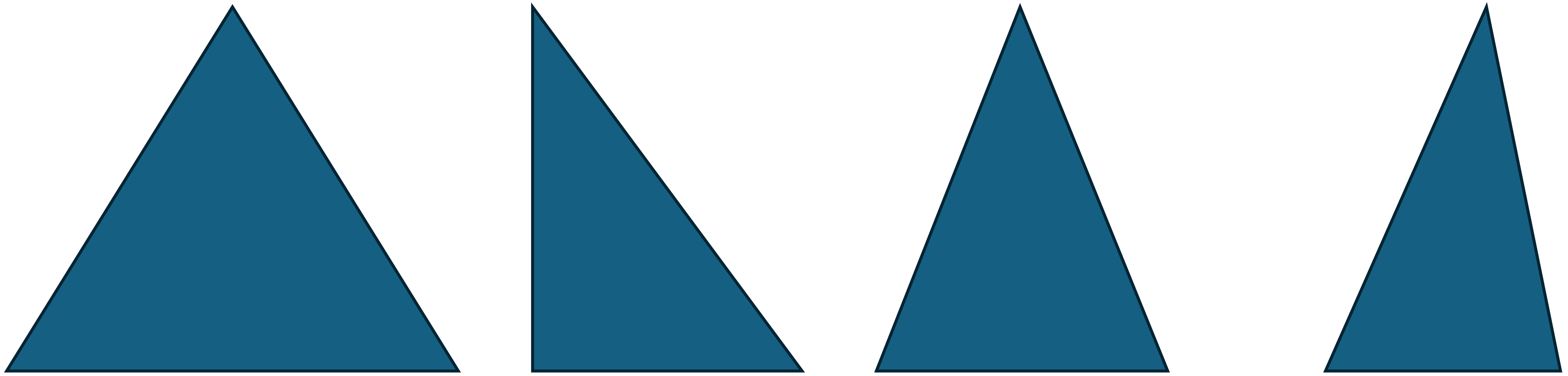
Triangles

Which two would
have the same area?



Triangles

Triangles are magical creatures because there is a relationship between the length of the sides and the number of degrees in the angles.



Equilateral
All angles same

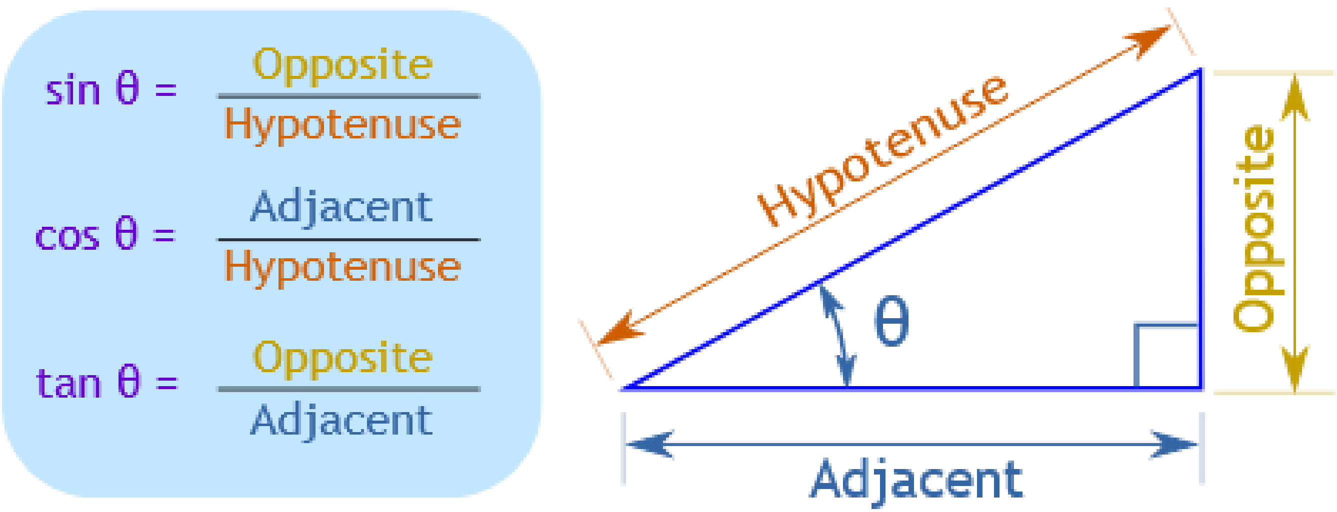
Right

Isosceles
2 angles same

Scalene
All angles different

Trigonometry

Right triangles are special: they can be either isosceles or scalene.
2 right scalenes back to back are isoscleles.



The two sides of a right triangle have a fixed relationship to the theta angle. The power of the right triangle is that if you have any two of these values, you can calculate the third.

Image source: https://www.mathsisfun.com/sine-cosine-tangent.html

Trigonometry

Trigonometry Table

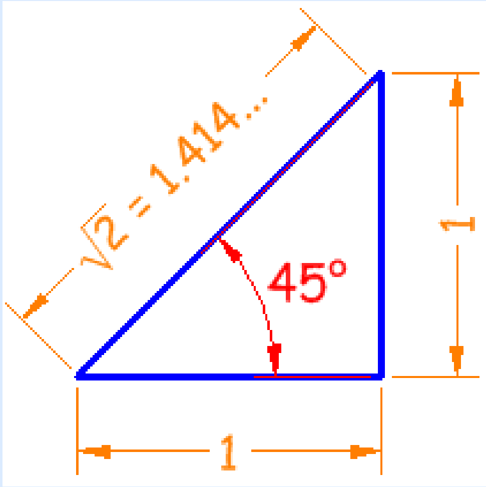
A	SIN(A)	COS(A)	Tan(A)
0	0.0000	1.0000	0.0000
1	0.0175	0.9998	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
9	0.1564	0.9877	0.1584
10	0.1736	0.9848	0.1763
11	0.1908	0.9816	0.1944
12	0.2079	0.9781	0.2126
13	0.2250	0.9744	0.2309
14	0.2419	0.9703	0.2493
15	0.2588	0.9659	0.2679
16	0.2756	0.9613	0.2867
17	0.2924	0.9563	0.3057

A	SIN(A)	COS(A)	Tan(A)
45	0.7071	0.7071	1.0000
46	0.7193	0.6947	1.0355
47	0.7314	0.6820	1.0724
48	0.7431	0.6691	1.1106
49	0.7547	0.6561	1.1504
50	0.7660	0.6428	1.1918
51	0.7771	0.6293	1.2349
52	0.7880	0.6157	1.2799
53	0.7986	0.6018	1.3270
54	0.8090	0.5878	1.3764
55	0.8192	0.5736	1.4281
56	0.8290	0.5592	1.4826
57	0.8387	0.5446	1.5399
58	0.8480	0.5299	1.6003
59	0.8572	0.5150	1.6643
60	0.8660	0.5000	1.7321
61	0.8746	0.4848	1.8040
62	0.8829	0.4695	1.8807

Image Source: https://sciencenotes.org/downloadable-trig-table-pdf/

Trigonometry

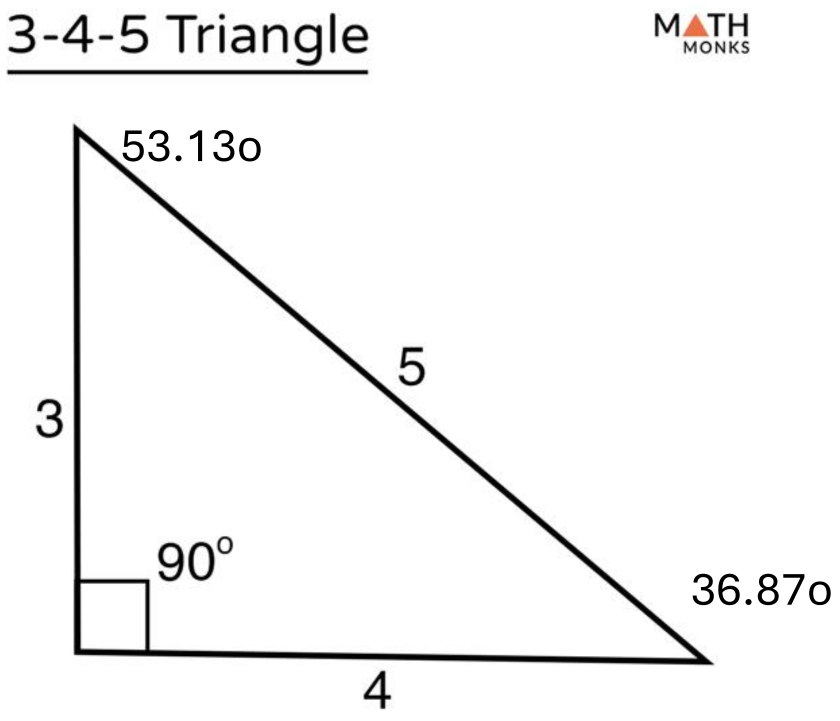
The classic 45° triangle has two sides of 1 and a hypotenuse of $\sqrt{2}$:



Sine	$\sin(45°) = 1 / 1.414 = 0.707...$
Cosine	$\cos(45°) = 1 / 1.414 = 0.707...$
Tangent	$\tan(45°) = 1 / 1 = 1$

Image source: https://www.mathsisfun.com/sine-cosine-tangent.html

Trigonometry



The theta angle for the sine of that pretty Pythagorean 3,4,5 right triangle is not a pretty number: not quite 37.

Image source: https://mathmonks.com/wp-content/uploads/2021/04/3-4-5-Triangle.jpg

Lifelong Learning in Second Life

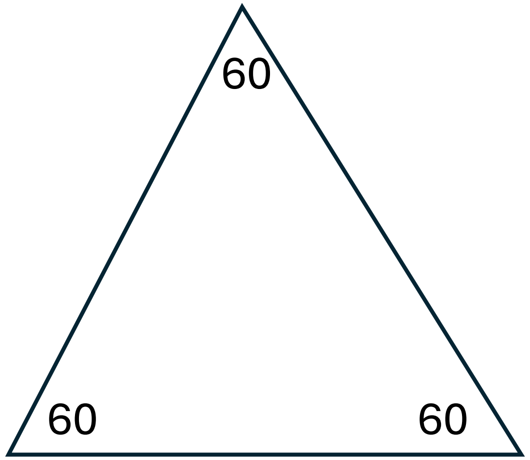
- I am not a mathematician. I am a biologist. I taught a nonmajors course here on Genome Island in Second Life for 14 years.
- While I was actively teaching, I didn't stray far from Genome Island, although I frequently visited the other sims in this wonderful cluster called the SciLands.
- But since I retired in 2022, I've been exploring some of the less formal educational opportunities in Second Life.
- One of those is Tagline's Math Club

Math Club? Me??

- My math background is very minimal. I took ONLY the math required for my major, which didn't, in those ancient days, go past trigonometry.
- I can add, subtract, divide and multiply. I can calculate a 20% tip. I can do my taxes. I can predict the progeny of a trihybrid cross.
- But some of my good friends are very good mathematicians, and here in SL they have been sharing their enthusiasm for math in informal activities like Math Club.
- And that is the Secret Superpower of Second Life. This is an amazing educational environment, and those opportunities are not restricted to people younger than 18.
- This is the story of how a math moron became obsessed with some properties of triangles.

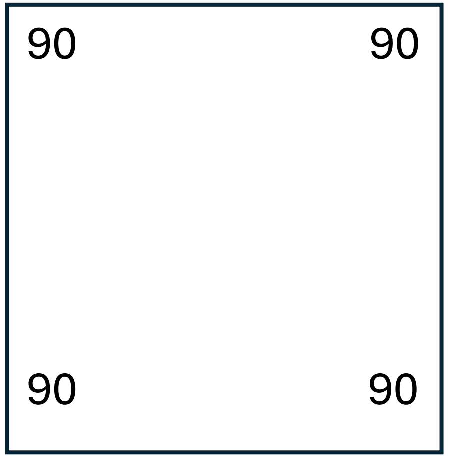
My obsession began here:

Triangle



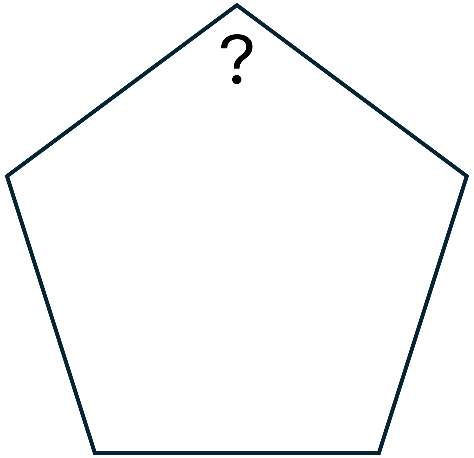
180

Square



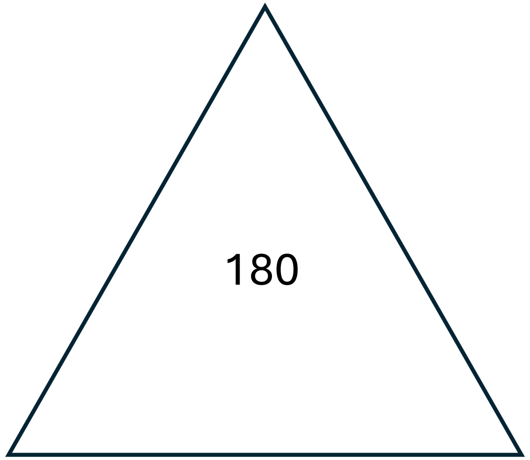
360

Pentagon

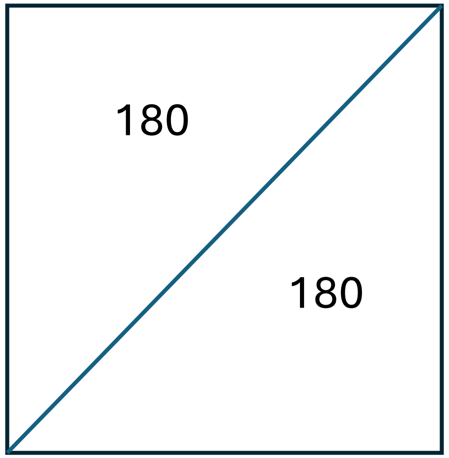


What number goes here?
In a regular pentagon, all of the corner values are the same.

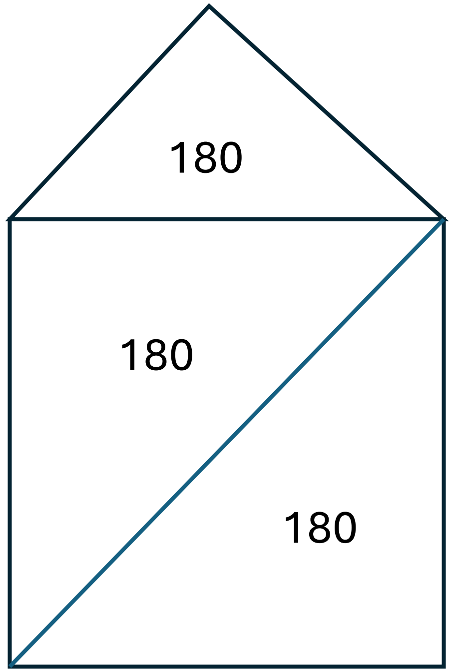
My obsession began here:



Triangle = 1 triangle
180
180/3 = 60



Rectangle = 2 triangles
360
360/4 = 90



Pentagon = 3 triangles
540
540 /5 = 108

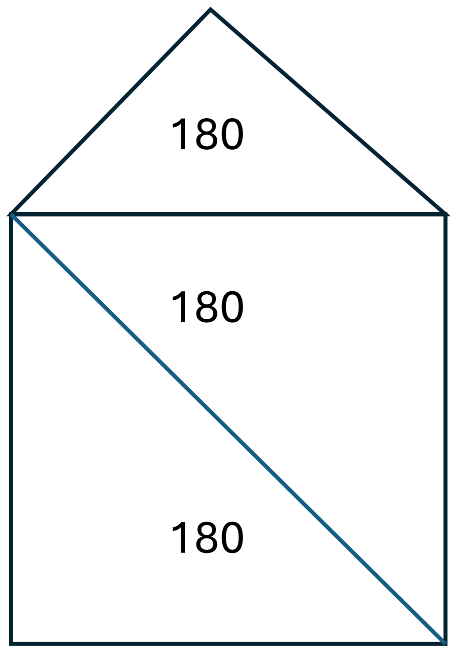
OMG!!!

Math moron discovers a math law!

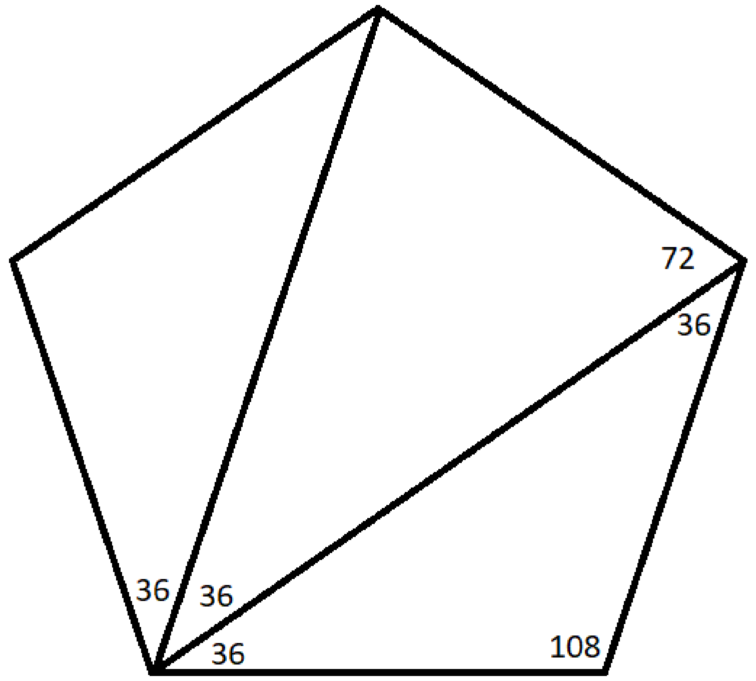
Every side you add to a polygon adds 180o to the total internal angle!

Polygons are made out of Triangles!

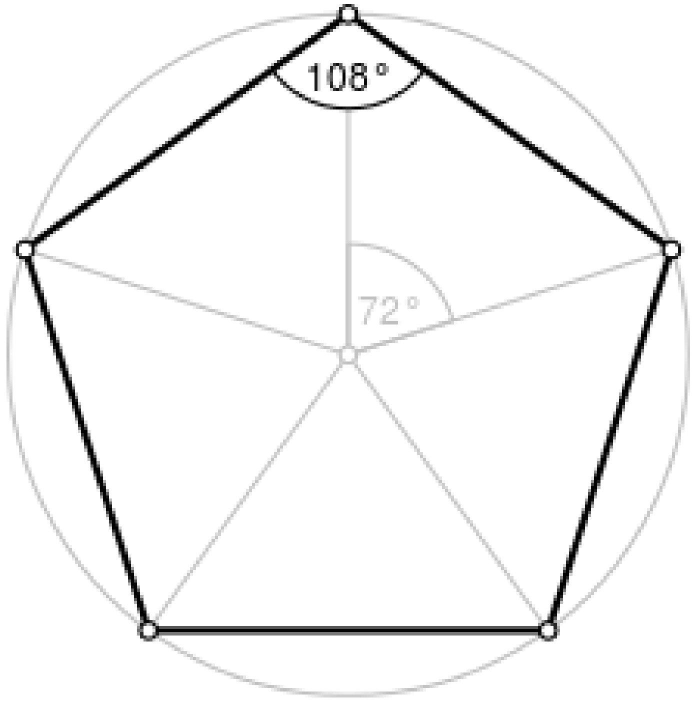
Wait a minute, why doesn't a pentagon have 5 triangles??? Depends on where you put them.



Irregular pentagon

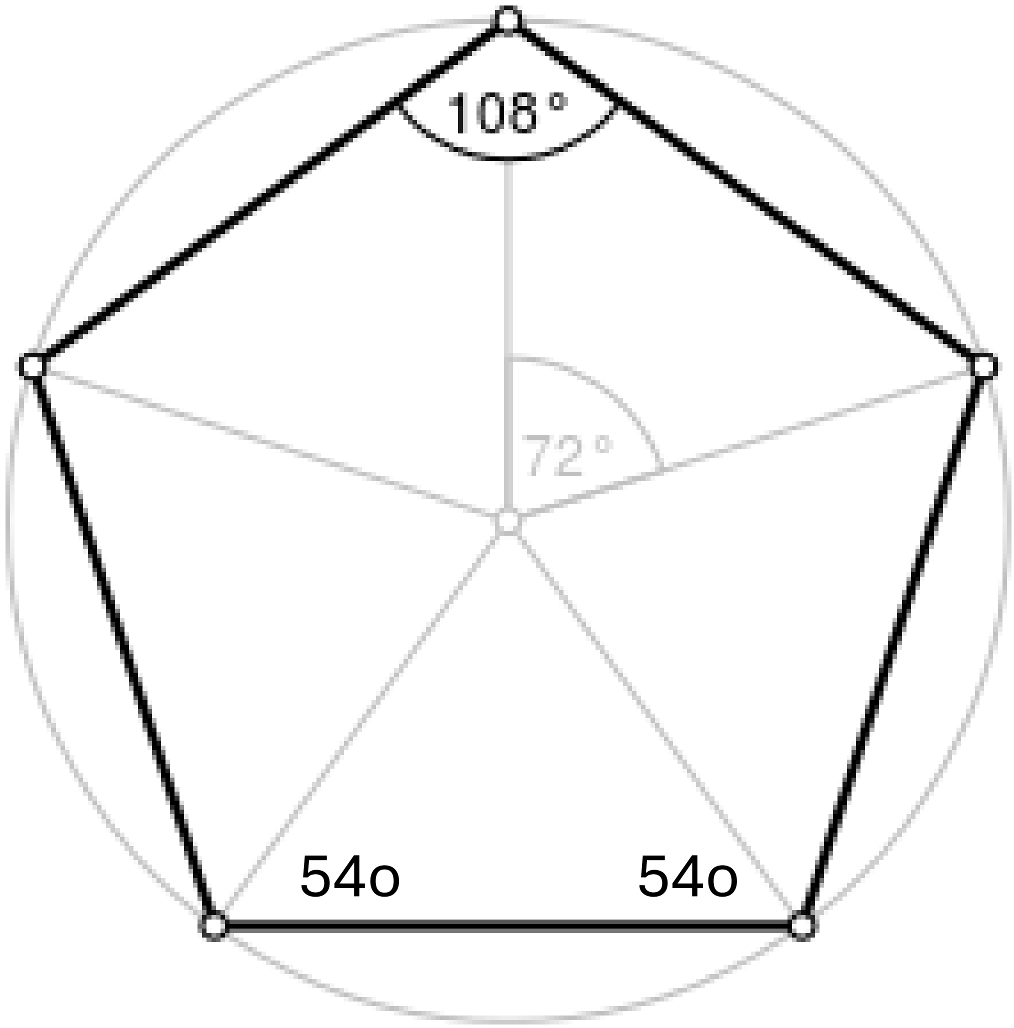


3-triangle pentagon



5-triangle pentagon

Pentagon angles

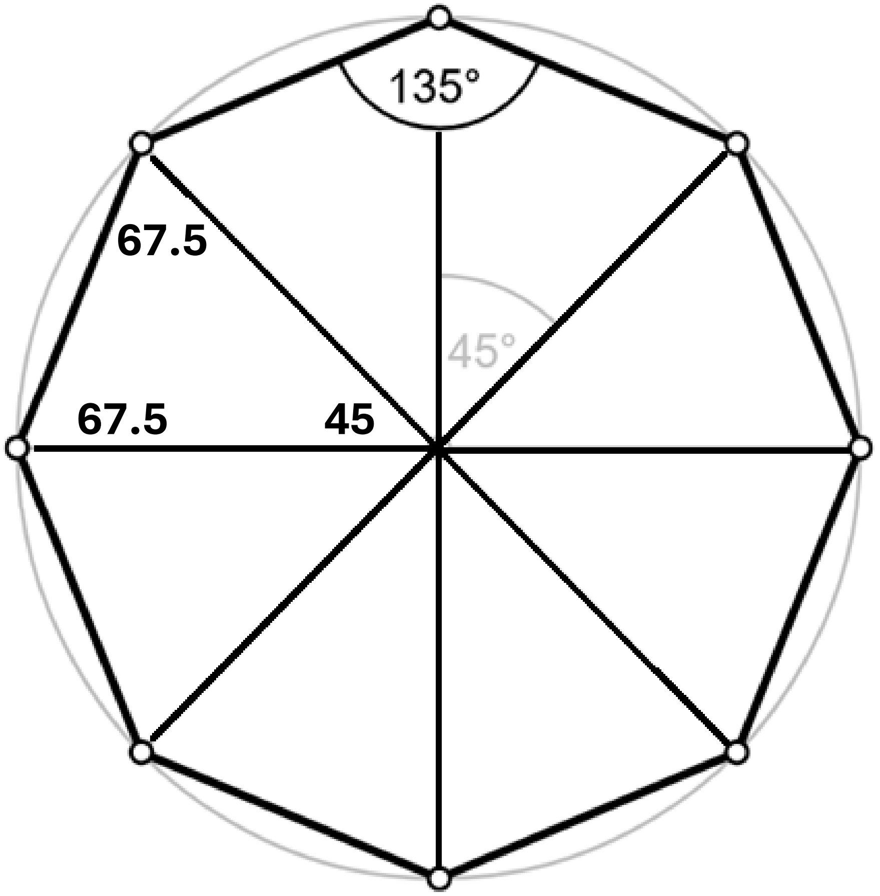


Typical polygons

In a typical polygon, the triangles are all the same (all isosceles except 1) and come together at a central point. Also, the number of triangles = the number of sides for the polygon.

In an atypical polygon, where did the angles from the two extra triangles go?

Internal angle = 135o

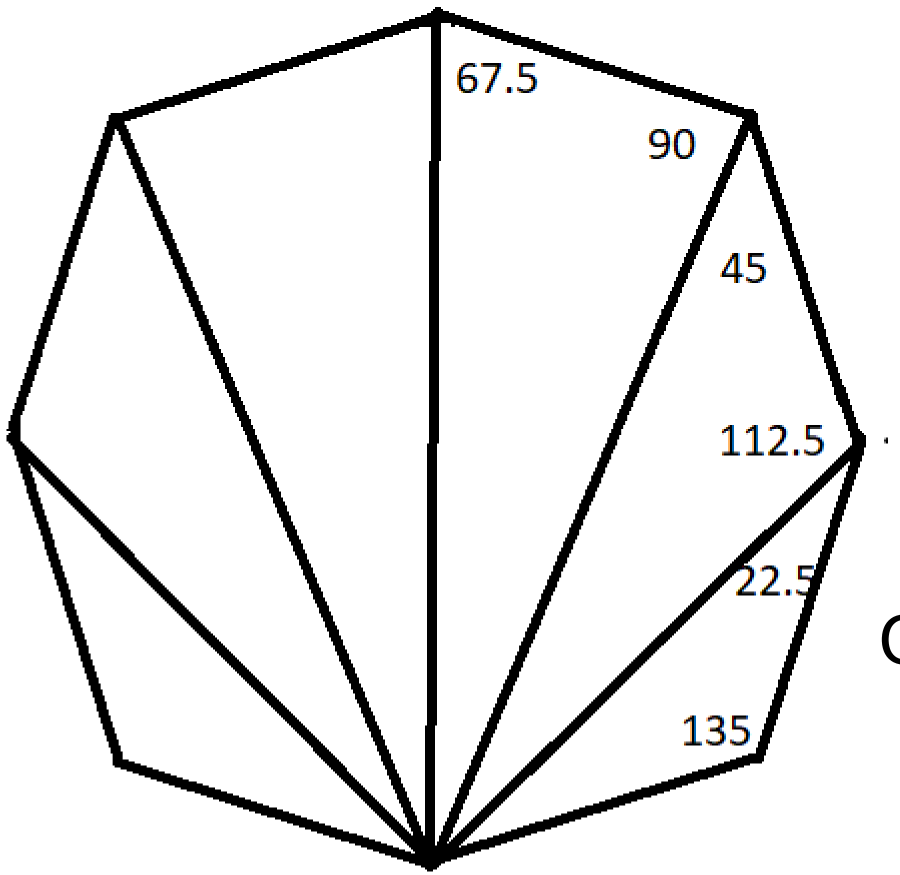


45 x 8
= 360o

8 Triangles

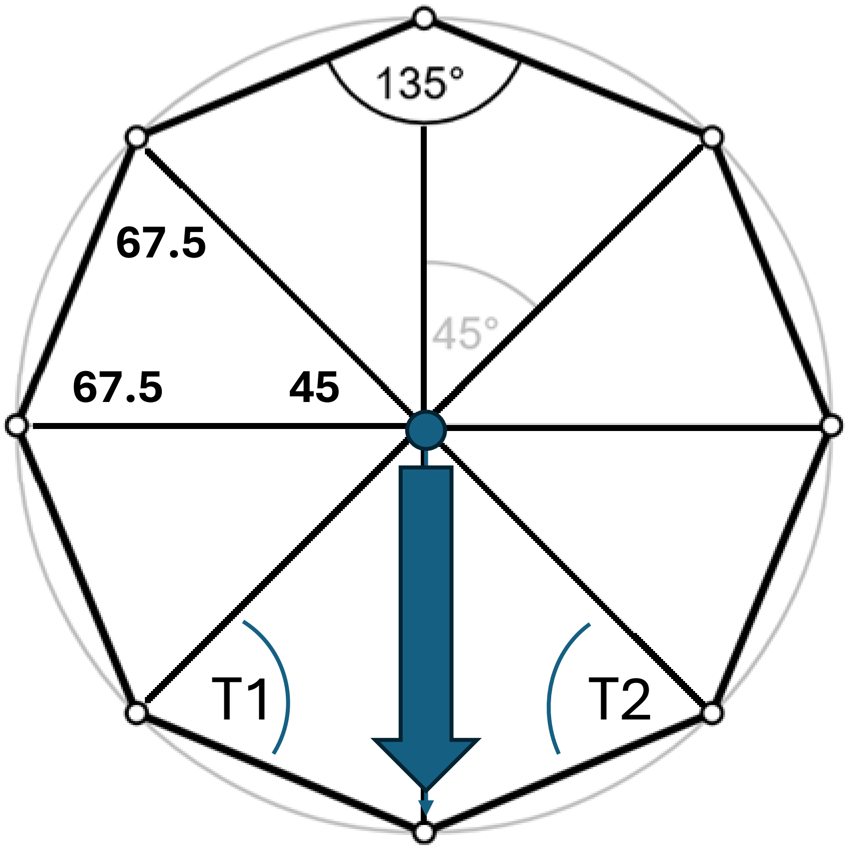
Polygon conversion

These two structures can be connected



8 sides and 6 triangles

Octagons

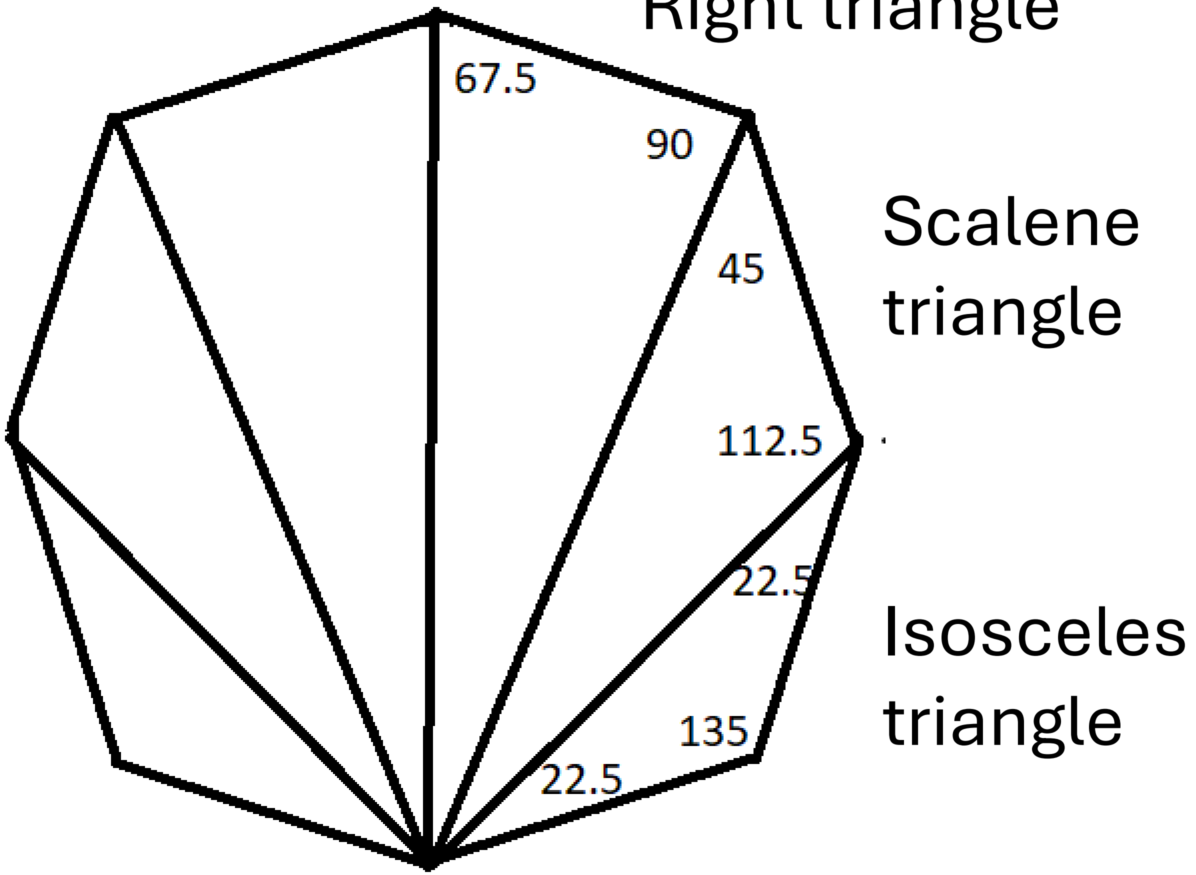


8 sides and 8 triangles

Atypical polygons

These polygons have all been subdivided by dividing them into triangles drawn from one edge point. This put all of the angles on the outside, so that the contributions of each triangle to the internal angles of the polygon could be seen. Several different triangles are involved.

Internal angle = 135o



6 Triangles

Other patterns then emerged

- The angles formed by the triangles follow a numerical pattern up and down both sides.
- There is always an obtuse isosceles triangle at opposite edges on both sides.
- The acute angles at the convergence point are always the same.

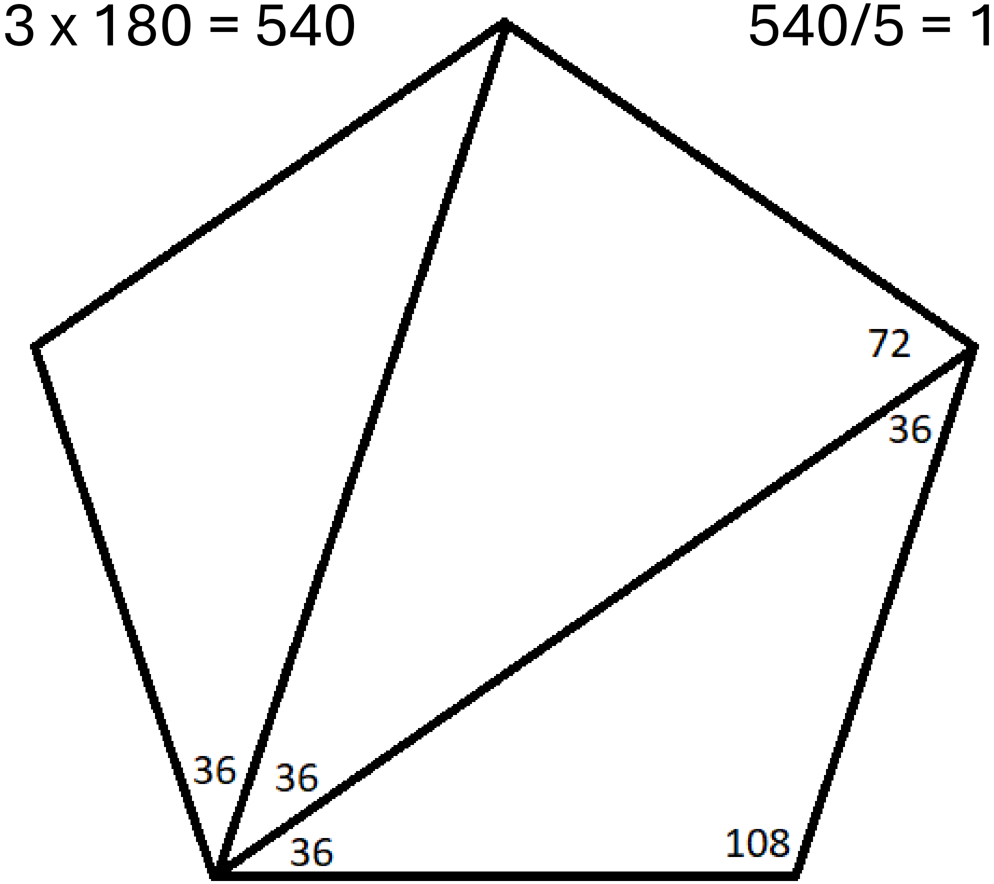
  AND

- In even- numbered polygons, there is always a right triangle at the center of the cluster.
- In odd-numbered polygons, there is always an isosceles triangle at the center of the cluster.

Pentagons and Hexagons

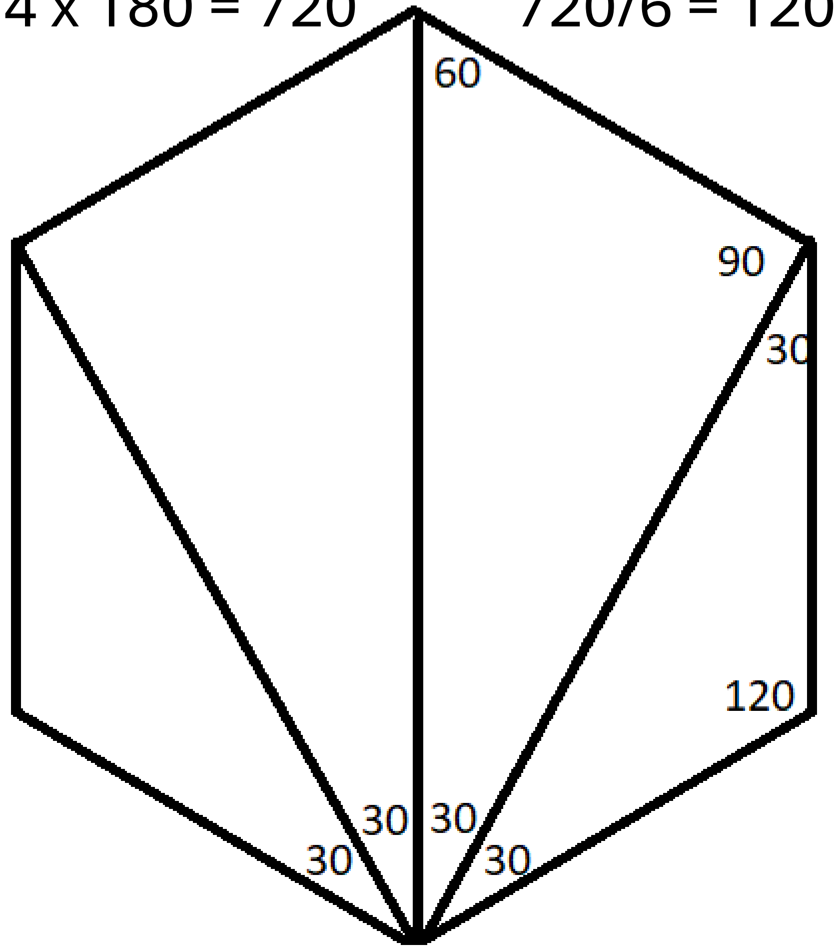
3 x 180 = 540

540/5 = 108



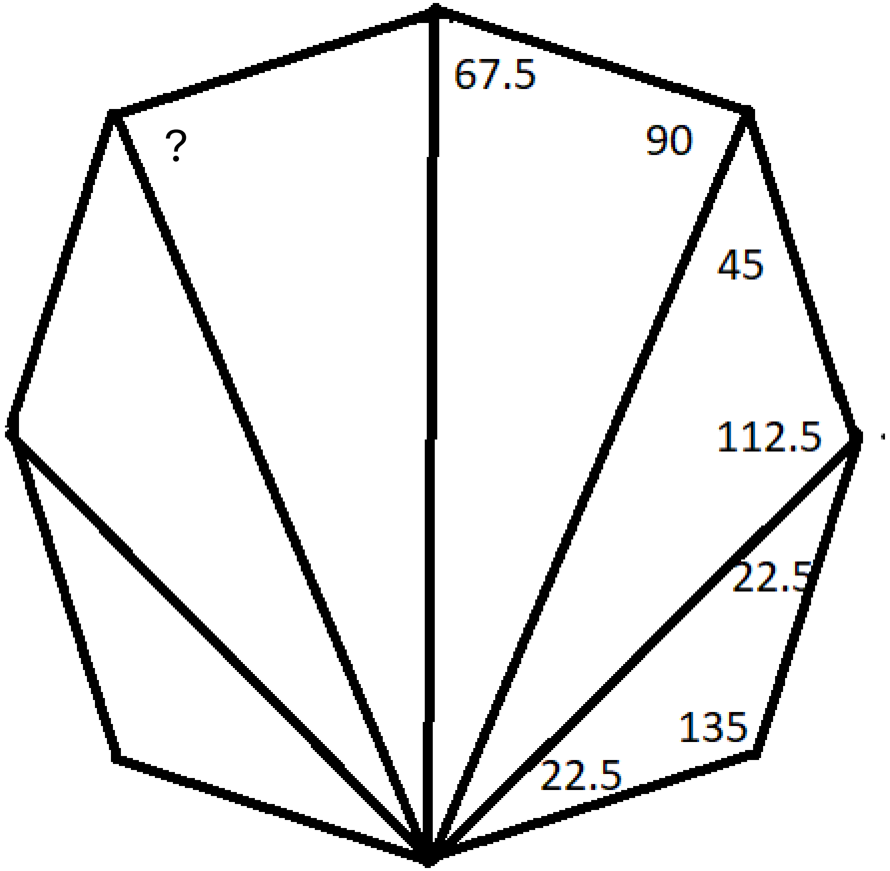
4 x 180 = 720

720/6 = 120

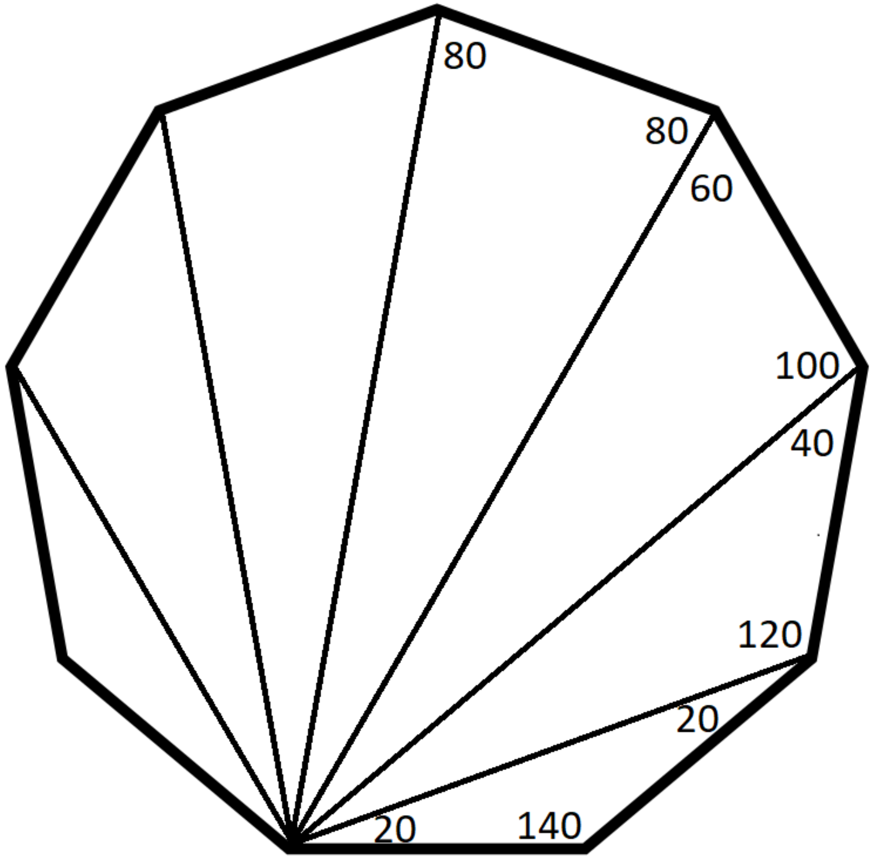


Octagons and Nonagons

6 x 180 = 1080 1080/8 = 135

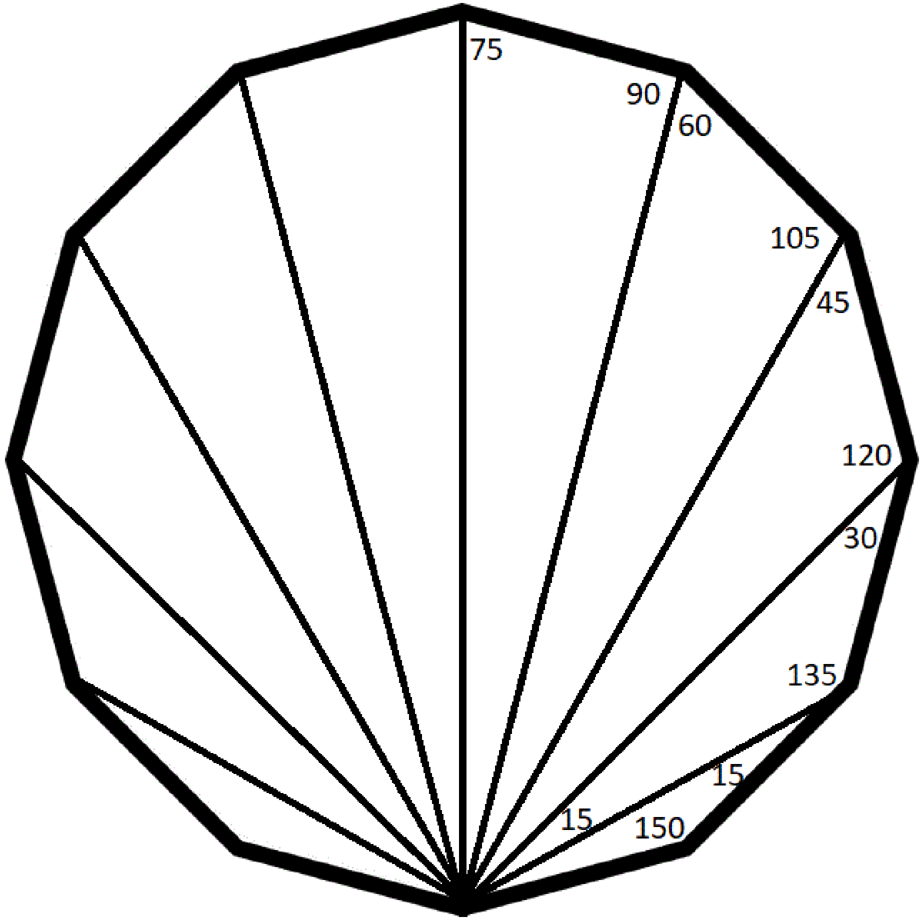


7 x 180 = 1260 1260/9 = 140

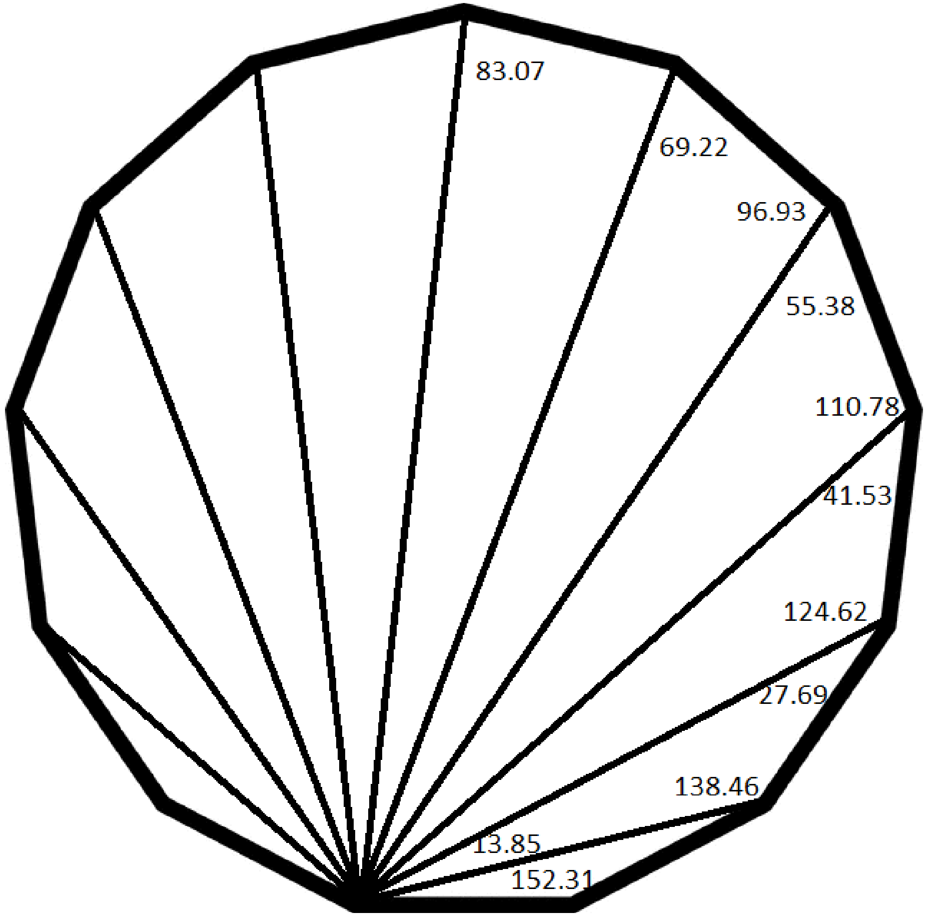


Dodecagons and Tridecagons

10 x 180 = 1800 1800/12 = 150

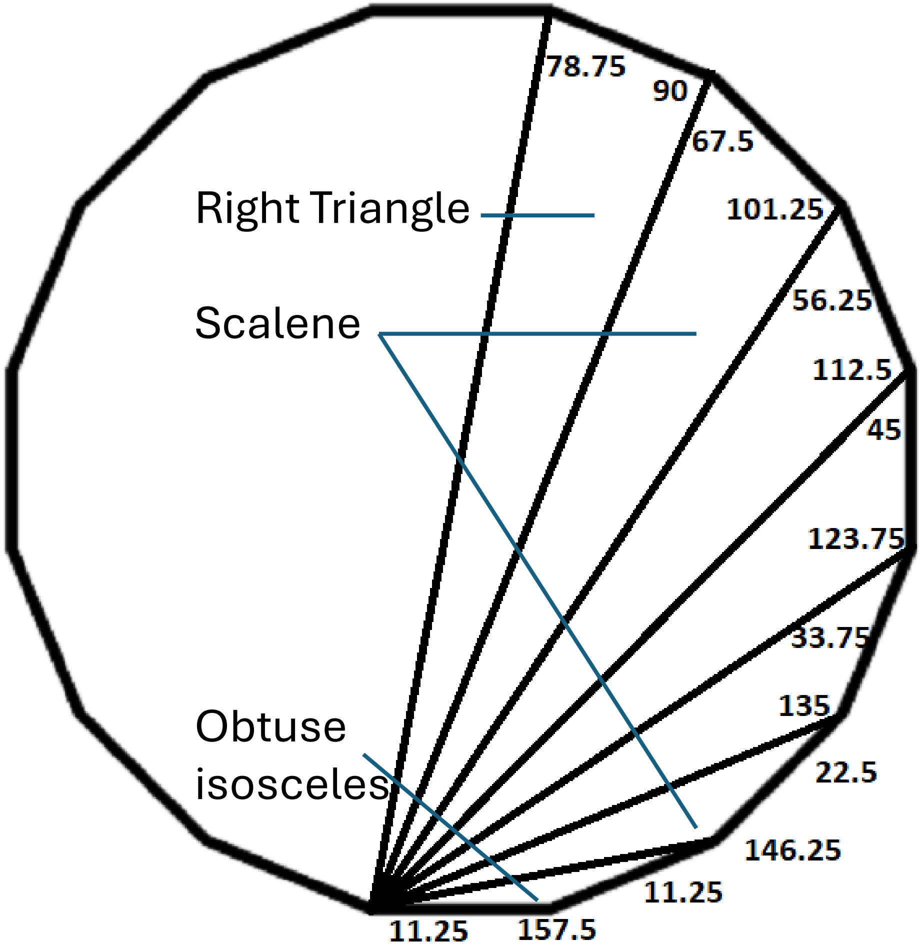


11x180 = 1980 1980/13 = 152.30769…



Hexadecagons

14x180 = 2520 2520/16 = 157.5



As we increase the number of triangles in the polygon, we are closing in on the relationship between the formula for calculating the area of a triangle and the formula for calculating the area of a circle....

Let’s think about 49 and a 50-sided polygon

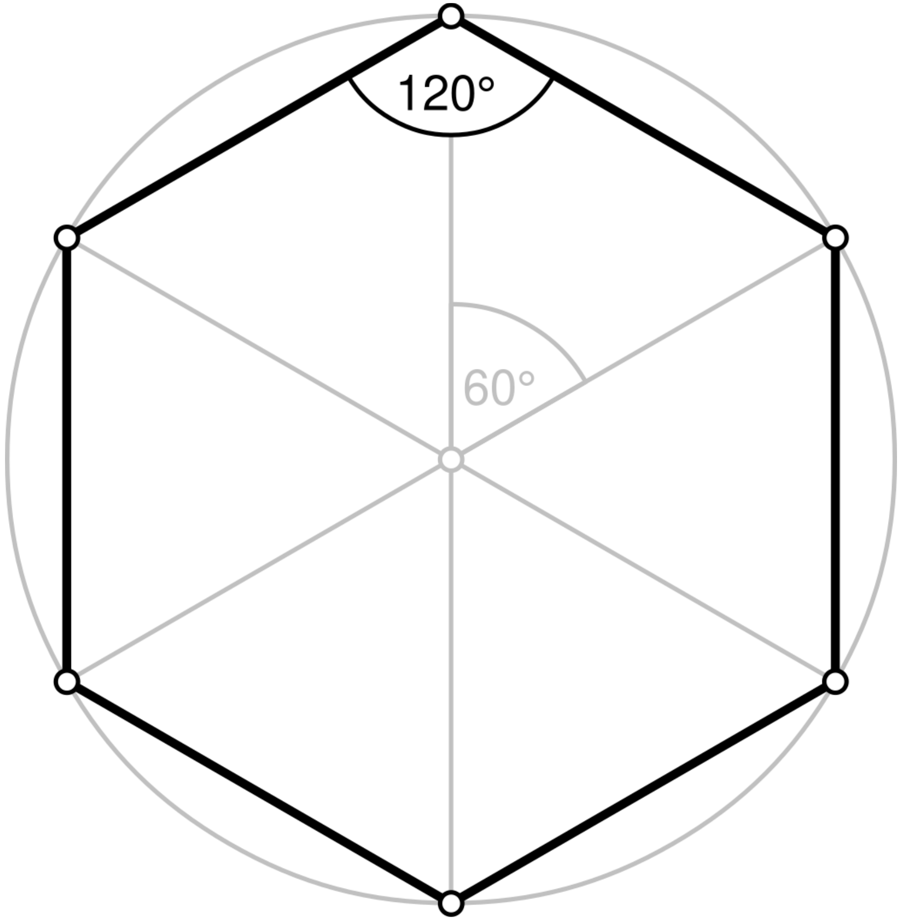
49-ogon (enneakaitessarakontadon)

47 triangles: 47 x 180 = 8460
Interior angle: 8460\49 = 172.6530612244898…
Acute angles at the base = 3.67347o
Base angle of central isosceles = 3.67347 x 24 = 88.16328

50-ogon (pentacontagon)

48 triangles: 48 x 180 = 8640
Interior angle: 8640/50 = 172.8
Acute angles at the base = 3.6o
Top angle at midpoint = 3.6 x 24 = 86.4.   And the next angle is….?

Circles and triangles



Area of a triangle: ½ x base x height

Area of a circle: πr^2

Circumference: $2\pi r$

Circumference = base of triangle

Radius (r) = height of triangle

½ x 2π r x r = ??

Image source: https://commons.wikimedia.org/w/index.php?curid=27295119

It’s Mathe-Magic!

Questions?