

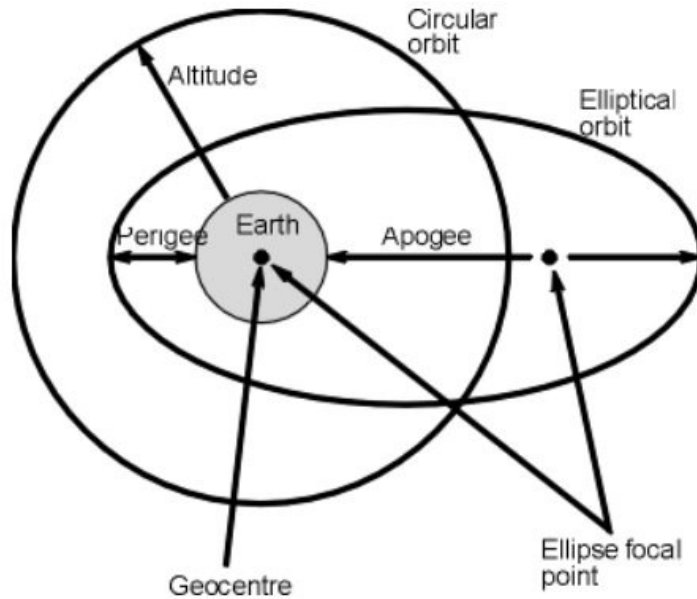
# Celestial Orbits

Syzygy Asymptote / Dr. William F. Wall\*

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Large Millimeter Telescope Alfonso Serrano, July 2018



Satellite orbits

<https://www.radio-electronics.com/info/satellite/satellite-orbits/satellites-orbit-definitions.php>



WFW, Cypress Falls Park, May 2018

\*Observational astronomer specializing in molecular clouds in nearby galaxies.

# *Outline*

Definitions

Motivation

I. Basic Physics

II. Orbital Basics and Real Orbits

- Basics
- Additional Physical Processes
- Examples of Real Orbits

Conclusions?



If I ran NASA, it would be mandatory for the ground crew to be dressed as apes when the space shuttle lands.

# *Definitions*

An orbit is the gravitationally curved trajectory of an object,[1] such as the trajectory of a planet around a star or a natural satellite around a planet. Normally, orbit refers to a regularly repeating trajectory, although it may also refer to a non-repeating trajectory. (<https://en.wikipedia.org/wiki/Orbit>).

# *Definitions*

Bodies are in orbit when they fall around each other due to their mutual gravity. (*W. F. Wall*)

@skydivespain #tracking #tracking #15000ft



5:55 AM - 15 Nov 2016

*Motivations:*

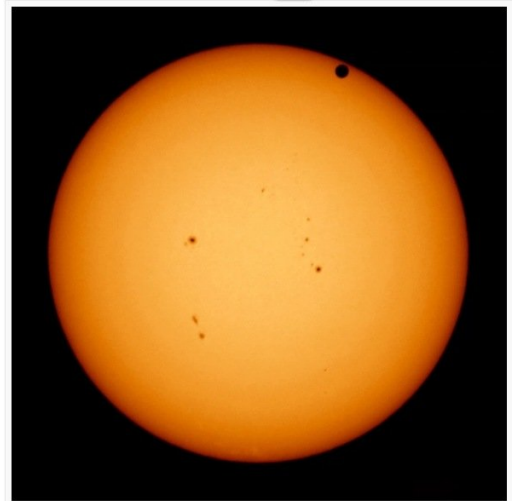
*Why are Orbits Interesting?*

# *Motivations:*

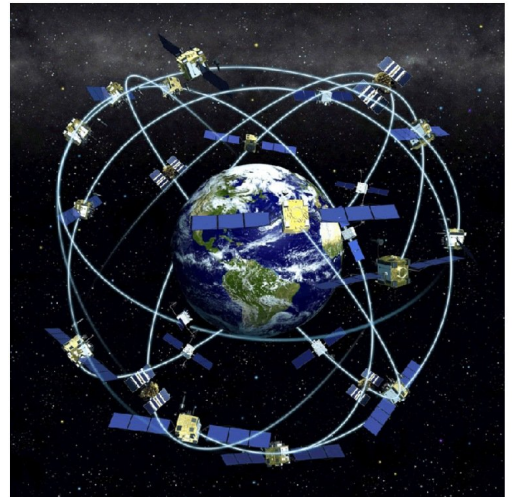
## *Why are Orbits Interesting?*

1) Days, seasons, years, tides, climate, heavenly motions and events (eclipses, occultations, transits, risings, settings).

2) Artificial satellites for probing the heavens, the Earth, for communications, for GPS. *Which have reshaped our worldwide civilization.*



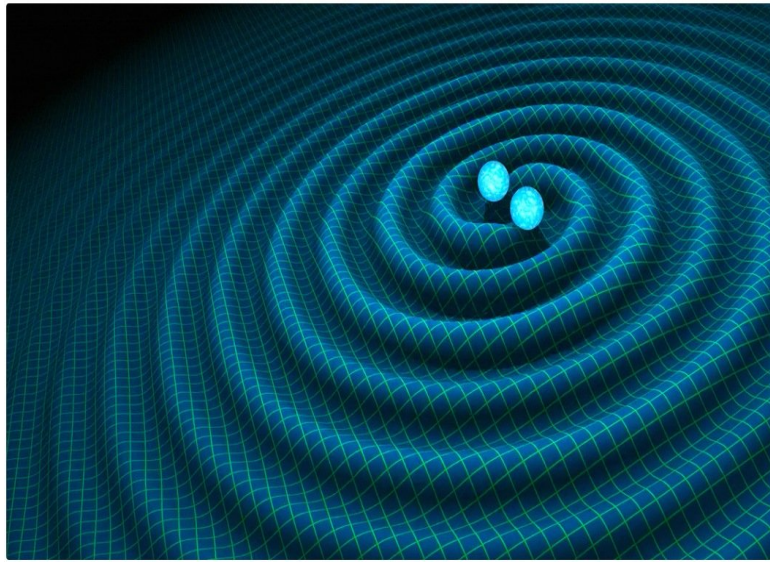
Transit of Venus – Image credit- Wikipedia –  
[http://en.wikipedia.org/wiki/File:2012\\_Transit\\_of\\_Venus\\_from\\_SF.jpg](http://en.wikipedia.org/wiki/File:2012_Transit_of_Venus_from_SF.jpg) – Venus is at the top right of the Sun's disk. Other dark blotches are sunspots.



The orbits of GPS satellites are inclined to the Earth's equator by about 55 degrees. The system is designed to ensure that at least four satellites are visible at least 15 degrees above the horizon at any given time anywhere in the world. (Image: © NOAA)



# *Motivations: Why are Orbits Interesting?*



Artist's conception of two black holes orbiting one another and producing gravitational waves. [Image Credit: NASA]

3) We can do kick-ass science!  
Exoplanets and gravitational waves, for example.



4) Space travel – exploration (solar system bodies and beyond), exploitation (mining moon and asteroids, inhabiting space).

# *Motivations:*

*Why are Orbits Interesting?*

## 5) ***Protection of the Earth from Collisions.***



The last days of dinosaurs during the Cretaceous Period, caused by a giant asteroid impact at Chicxulub off the coast of Mexico. | Stocktrek Images via Getty Images



# *I. Basic Physics:*

# *I. Basic Physics:*

- Newton's laws of motion
- Newtonian gravity
- Weight vs Mass
- Velocity & Acceleration
  - Linear acceleration
  - Centripetal acceleration
- Fictitious vs True Forces
- Velocity in a circular orbit
- Work and Energy
  - Potential Energy
  - Potential
  - Kinetic Energy
- Total Energy
  - Escape velocity
  - Orbital trajectories

# *Newton's Laws of Motion*

- 1) A body in motion stays in motion and a body at rest stays at rest unless acted upon by an external unbalanced force. (*Inertia*)
- 2) Applying such a force to a body results in an acceleration,  $\vec{a}$ , that is proportional to the force,  $\vec{F}$ , and inversely proportional to the body's mass,  $m$ .

$$\vec{F} = m \vec{a}$$

$$W = m g$$

Weight = mass X acceleration due to gravity

# Newton's Laws of Motion

3) For every action there is an equal and opposite reaction.



Momentum,  $\vec{p} = m \vec{v}$ , is conserved.

### Angular Momentum of a Particle

The angular momentum is the same at every point on an orbit. When it is closer, it increases speed.

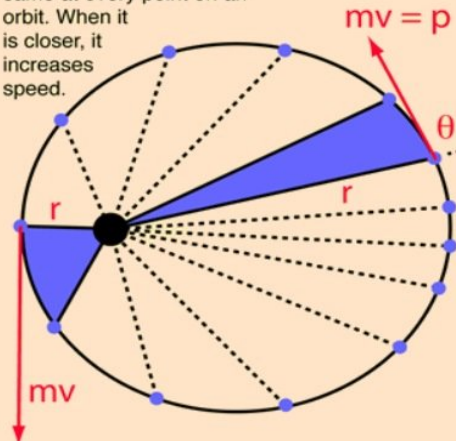
The angular momentum of a particle of mass  $m$  with respect to a chosen origin is given by

$$L = mvr \sin \theta$$

or more formally by the vector product

$$L = \vec{r} \times \vec{p}$$

The direction is given by the right hand rule which would give  $L$  the direction out of the diagram. For an orbit, angular momentum is conserved, and this leads to one of Kepler's laws. For a circular orbit,  $L$  becomes

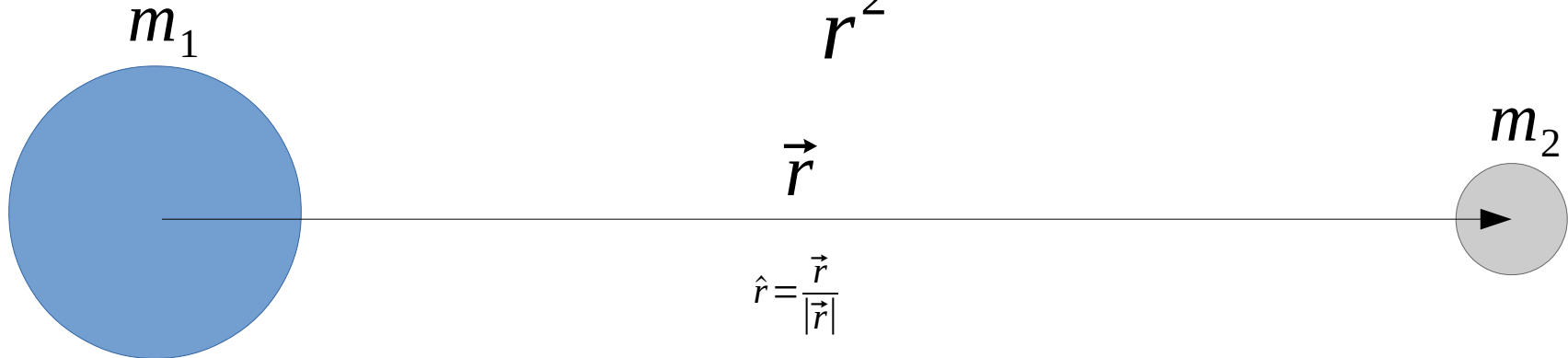
$$L = mvr$$


<http://hyperphysics.phy-astr.gsu.edu/hbase/amom.html>

# *Newton's Laws of Motion*

- 4) The gravitational attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them.

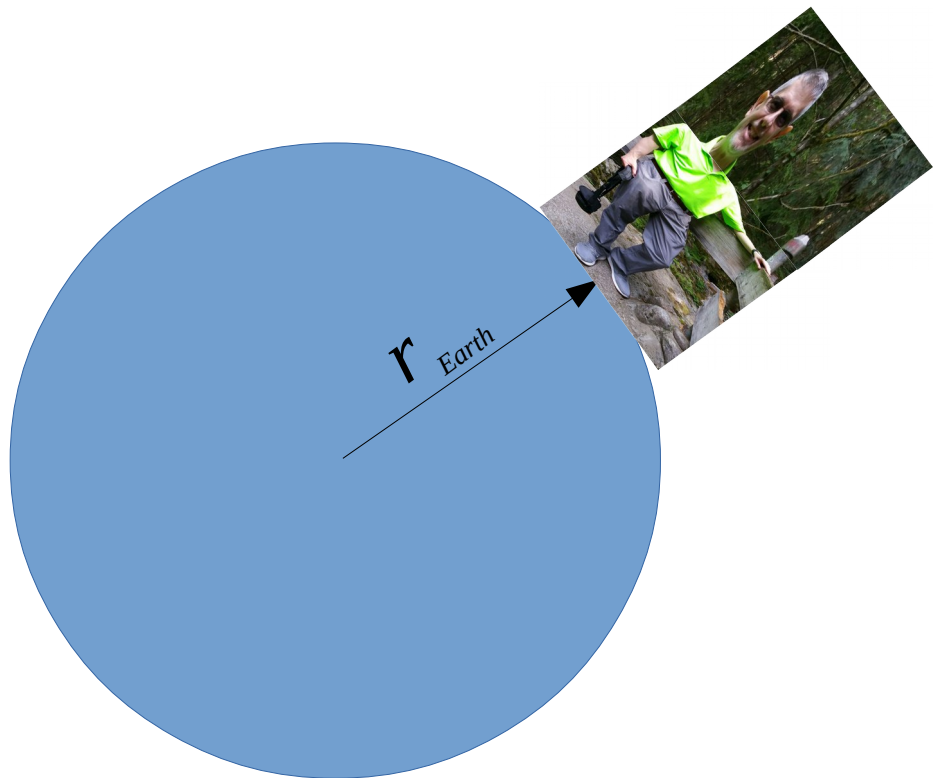
$$\vec{F} = -\frac{G m_1 m_2}{r^2} \hat{r}$$





# *Newton's Laws of Motion*

Let's have fun with Newton's laws. We'll combine #2 with #4:



$$W = F$$

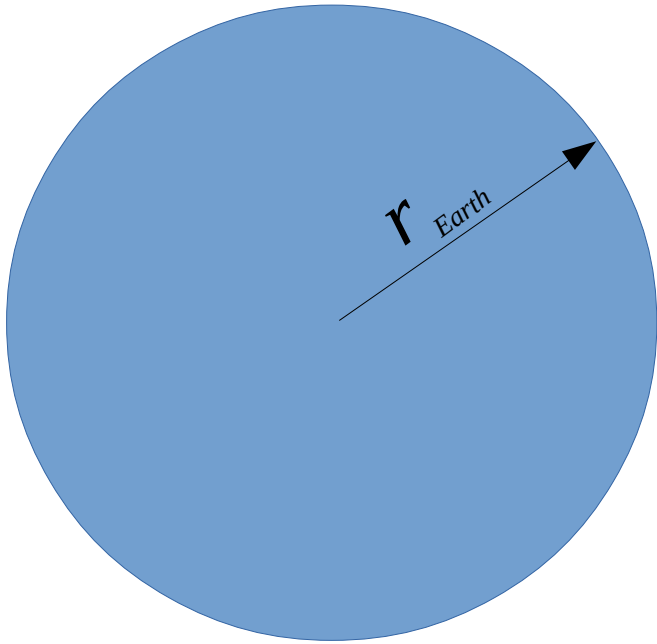
*Weight = Force of gravity*

$$m_{\text{astronomer}} g = \frac{G M_{\text{Earth}} m_{\text{astronomer}}}{r_{\text{Earth}}^2}$$

*Law #2 = Law #4*

# *Newton's Laws of Motion*

Let's be **shocked** by Newton's laws when we combine #2 with #4!!



$$g = \frac{G M_{\text{Earth}}}{r_{\text{Earth}}^2}$$

***The astronomer is irrelevant!!***

Important consequence of the above is that *all free falling bodies accelerate at the same rate as they fall regardless of their masses.*

***Feather will free fall like a stone!*** (WHEN there's no air.)

Free falling = accelerating freely due to gravity with *NO* additional forces present. (*NO* air resistance.)

I realized after taking this photo that I could use it to estimate the radius of the Earth! But I did not have an accurate measure of the camera's height above the lake surface, i.e., 4 to 10 m.



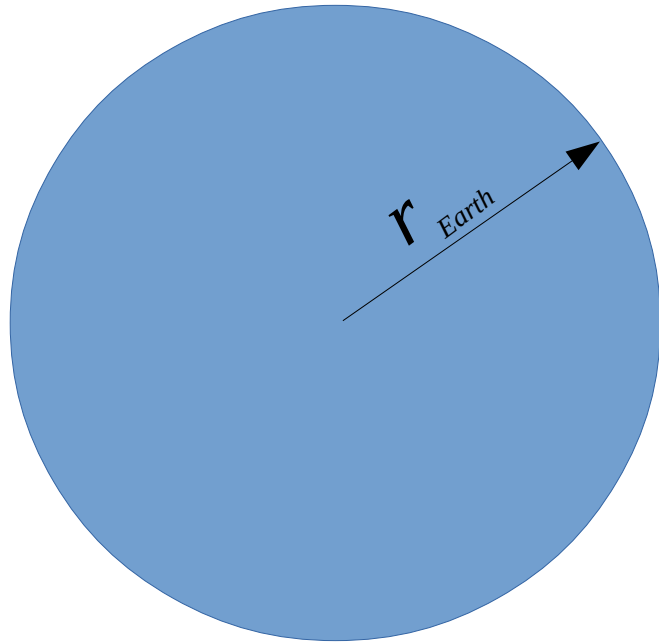
Consequently, my estimate of  $r_{\text{Earth}}$  was only about 11% accurate: 5930 to 7060 km. (Correct number is 6371 km.)

Toronto visible across Lake Ontario from Niagara-on-the-Lake, 2009, WFW

# Newton's Laws of Motion

Let's be *enlightened* by Newton's laws when we combine #2 with #4!!

The quantities  $g$ ,  $G$ , and  $r_{\text{Earth}}$  have been measured on the Earth.



*Six sextillion tonnes!* (American)  
*Six thousand trillion tonnes!* (British)

$$g = \frac{G M_{\text{Earth}}}{r_{\text{Earth}}^2}$$

Using  $g = 9.807 \text{ m s}^{-2}$ ,

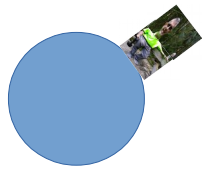
$$G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2},$$

$$\text{and } r_{\text{Earth}} = 6.371 \times 10^6 \text{ m}$$

$$M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

OR about

$$6 \times 10^{21} \text{ tonnes}$$



# *Mass versus Weight*



*Système Internationale (SI)*

## Person on Earth

$$g = 9.807 \text{ m s}^{-2}$$

$$\text{Mass} = 60 \text{ kg}$$

$$\text{Weight} = 588 \text{ Newtons (N)}$$

*British Imperial*

$$g = 32.17 \text{ ft s}^{-2}$$

$$\text{Weight} = 132 \text{ lb}$$

$$\text{Mass} = 4.11 \text{ slugs}$$

## Person on the Moon

$$g = 1.625 \text{ m s}^{-2}$$

$$\text{Mass} = 60 \text{ kg}$$

$$\text{Weight} = 97.5 \text{ N}$$

$$g = 5.3313 \text{ ft s}^{-2}$$

$$\text{Weight} = 21.9 \text{ lb}$$

$$\text{Mass} = 4.11 \text{ slugs}$$



# *Mass versus Weight*

**Mass** = *That property of matter that manifests itself as inertia. Or the amount of matter in a body.*

**Weight** = *The force with which gravity pulls on a body. Or how hard it presses down on a level surface.*

# *Velocity & Acceleration*

Both are vector quantities having magnitude and direction, but their directions can be distinct.

*Only* if the **acceleration** has a non-zero projection onto the **velocity** vector will it change the magnitude of the velocity vector (**speed**) of the moving body.

# Velocity & Acceleration

CASE 1: Velocity and acceleration vectors are parallel. Speed *increases*, direction is constant.  
Conventionally stated as “the body is accelerating.”

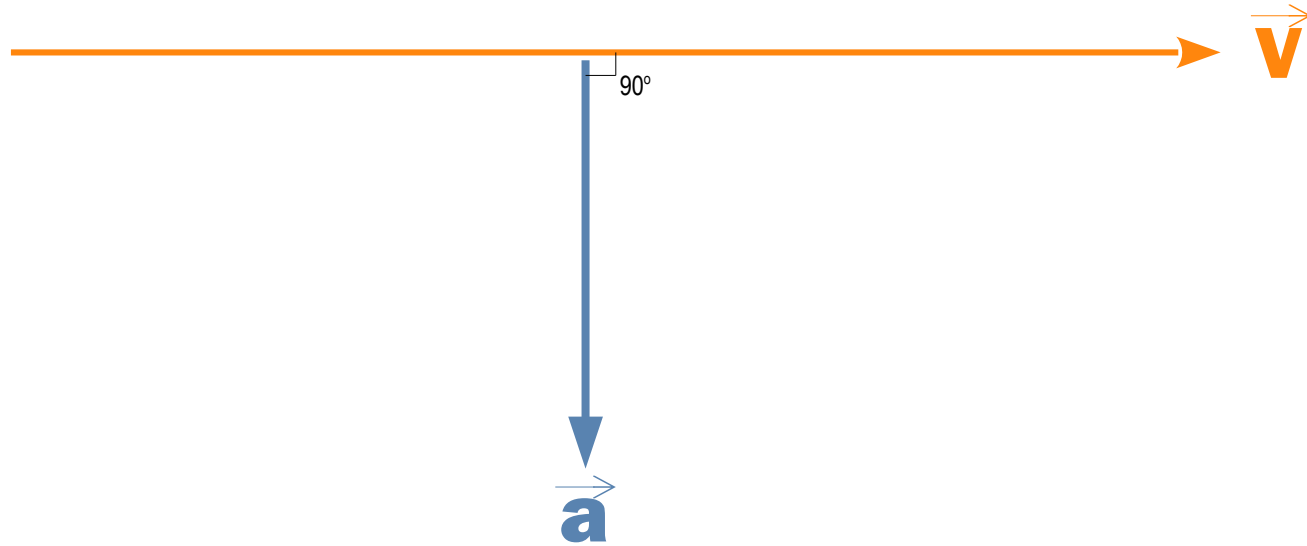


CASE 2: Velocity and acceleration vectors are *anti*-parallel. Speed *decreases*, direction is constant.  
Conventionally stated as “the body is *decelerating*.”



# Velocity & Acceleration

CASE 3: Acceleration is *perpendicular* to velocity. Speed is constant, direction changes at constant angular velocity,  $\omega$ , resulting in *circular motion*.



# Velocity & Acceleration



At the top, the speed is zero, but acceleration is still 1 g.

Upward V with speed decreasing. (CASE 2)

Downward acceleration is constant at 1 g

Downward V with speed increasing. (CASE 1)

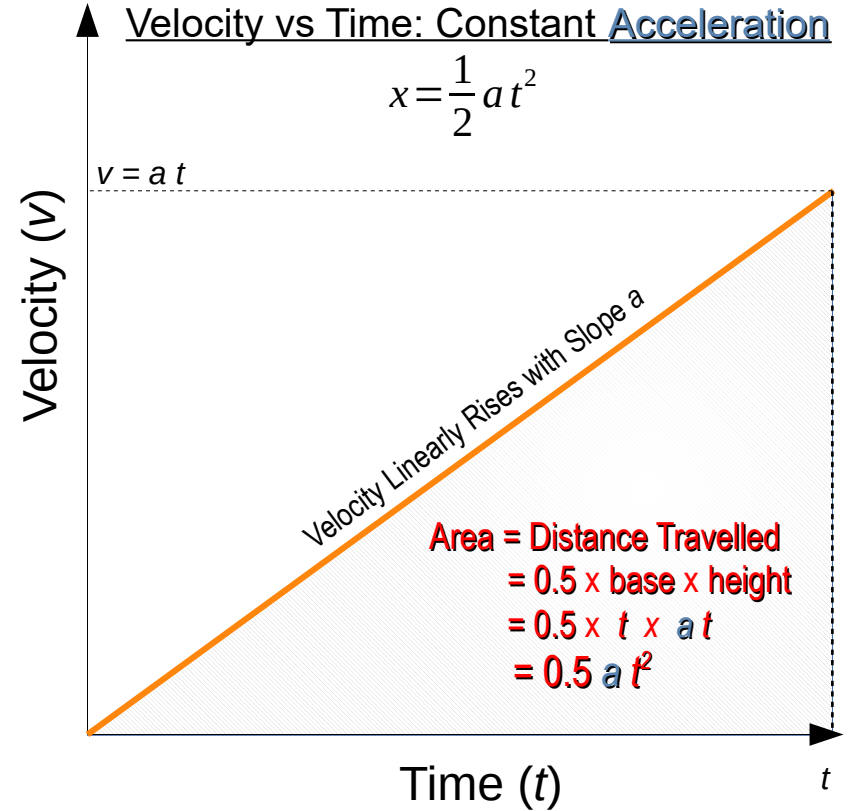
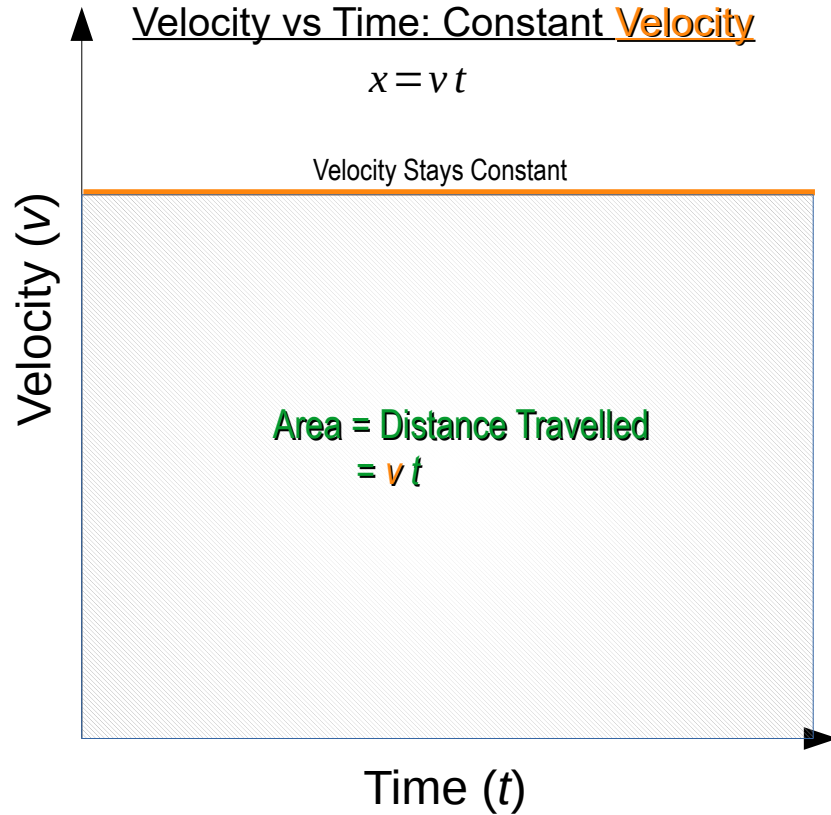
Tossing a ball up and down demonstrates Case 2 and Case 1. The velocity keeps changing, but the acceleration is constant at 1 g (assuming no air resistance).

Repeat the experiment with the Earth shrunk down to a point and the ball is now on a radial *orbit* (i.e., zero angular moment).



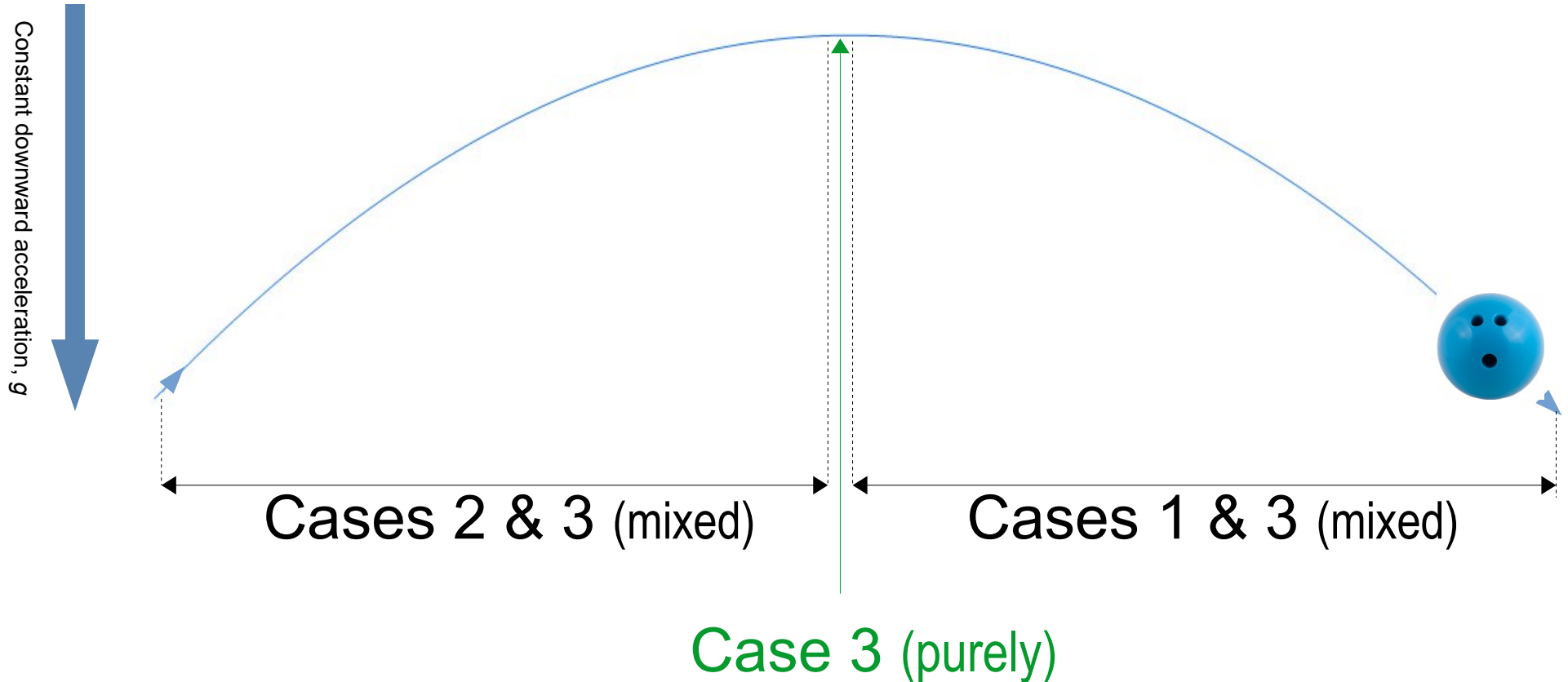


# Velocity & Acceleration



# Velocity & Acceleration

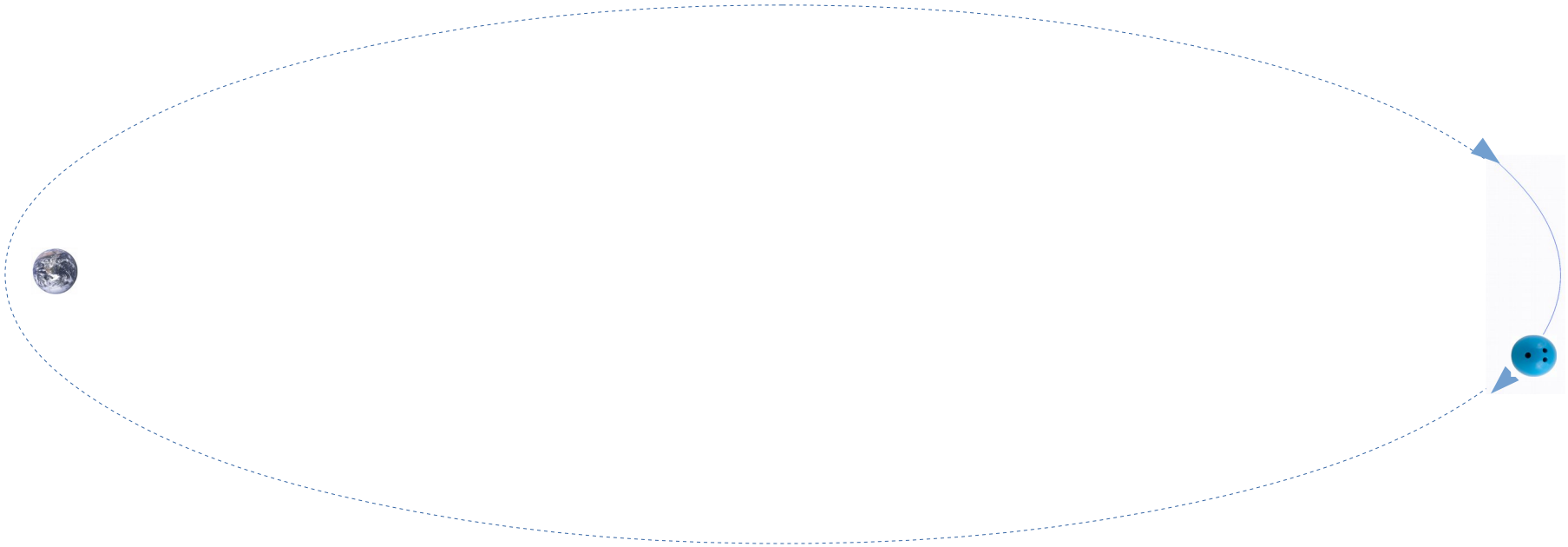
Projectile Launched\* near Earth's Surface: Follows parabolic path in the absence of air resistance.



\* Ball thrown.

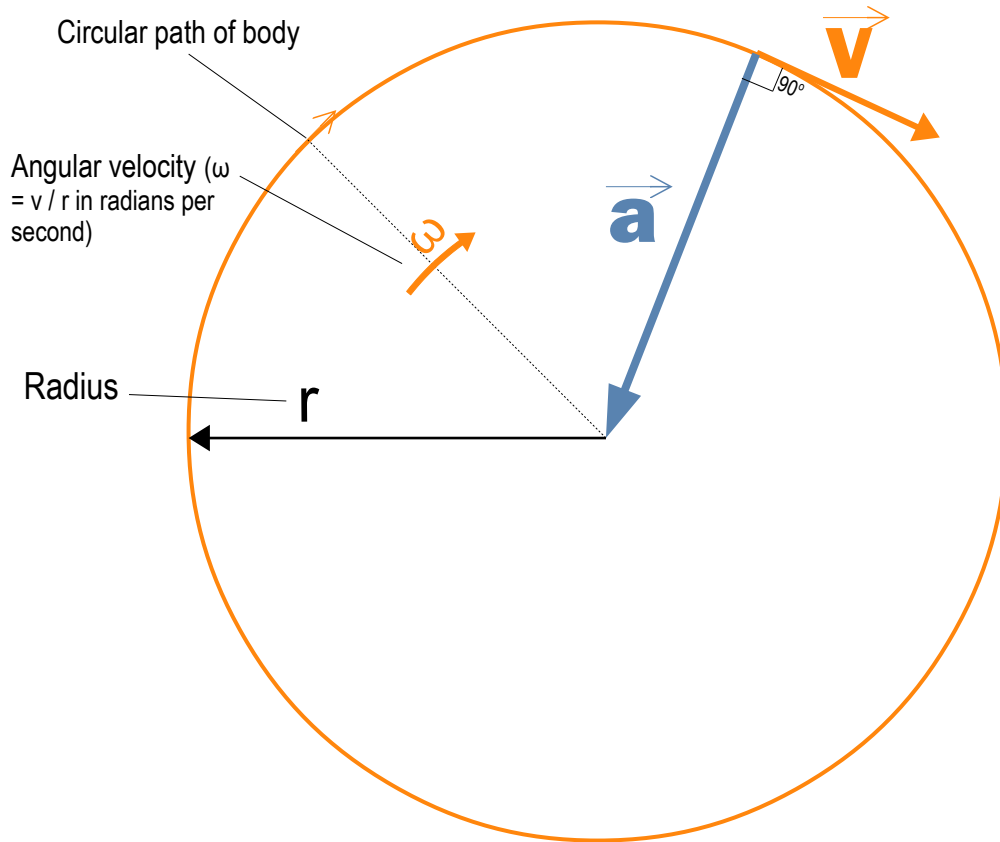
# *Velocity & Acceleration*

That same parabolic path is really part of a highly eccentric ellipse that represents an orbit\* around the Earth's centre (visible if the Earth were shrunk down).



\* Next time you throw a ball, remember that you're putting that ball into a temporary (decaying) highly eccentric orbit about the Earth's centre.

# Velocity & Acceleration



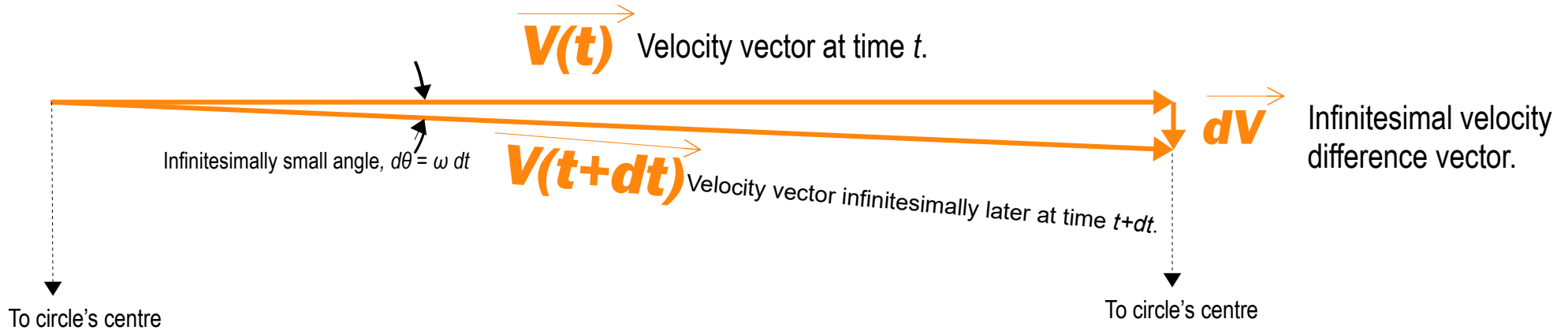
CASE 3: Body is constantly accelerating towards the centre of the circle, i.e., undergoing *centripetal acceleration* (which occurs because of some acting *centripetal force*).

The body never reaches that centre because of the tangential velocity.

This is what occurs in a circular orbit of one celestial body around another.

# Velocity & Acceleration

Centripetal Acceleration of Body on Circular Path:



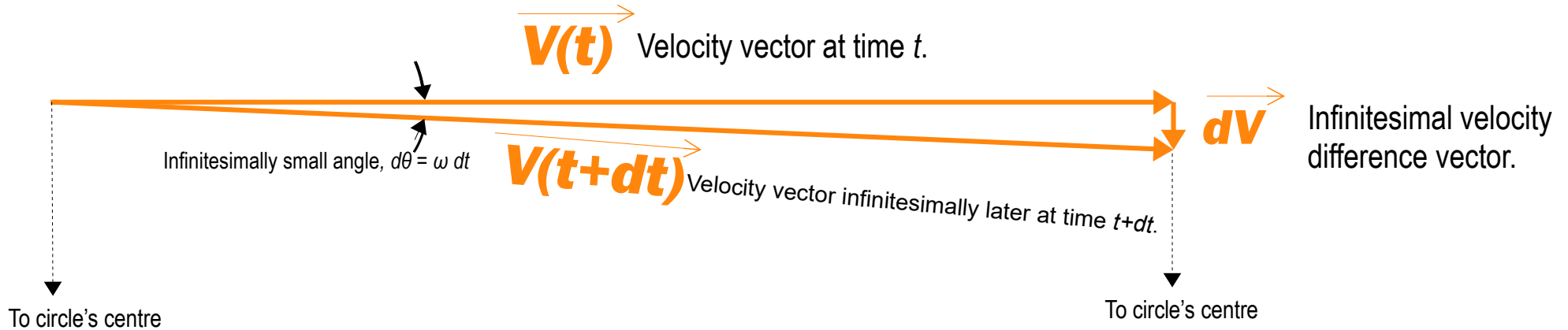
The velocity vectors above are like a thin wedge of pie. So,  $dV$  is like an infinitesimal arc. Accordingly,

$$dV = V d\theta$$

where  $d\theta$  is in radians and  $dV$  and  $V$  are the magnitudes of the velocity and velocity difference vectors, respectively.

# Velocity & Acceleration

Centripetal Acceleration of Body on Circular Path:



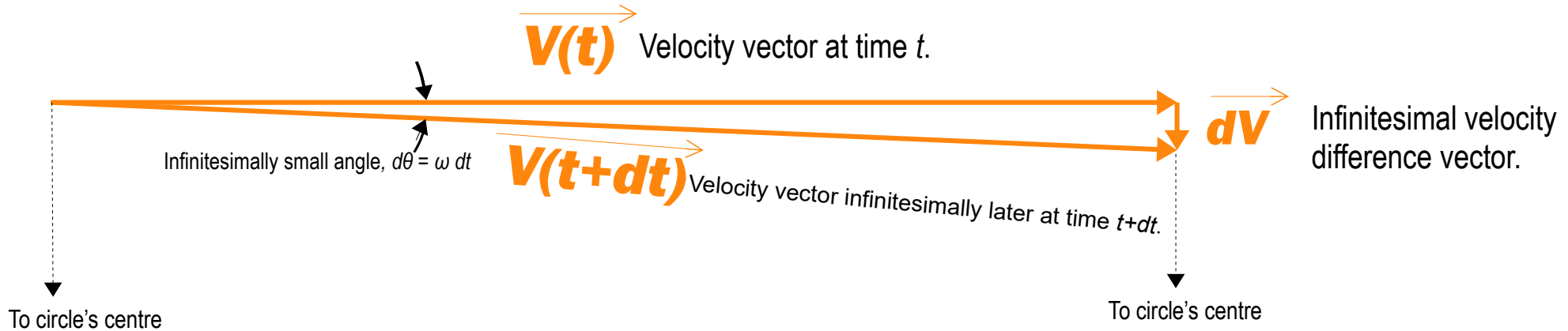
Using  $d\theta = \omega dt$  and  $\omega = V/r$  and remembering that  $d\vec{V}$  is really a vector pointing at the circle's centre:

$$\frac{d\vec{V}}{dt} = -\frac{V^2}{r} \hat{r} ,$$

which is the rate of change of velocity, or acceleration.

# Velocity & Acceleration

Centripetal Acceleration of Body on Circular Path:



This acceleration is the centripetal acceleration,  $a_c$ , so,

$$\vec{a}_c = -\frac{V^2}{r} \hat{r}$$

or

$$\vec{a}_c = -\omega^2 r \hat{r} .$$



# Velocity & Acceleration

## Centripetal Acceleration of Body on Circular Path:

- Applying centripetal acceleration formula to real situations.
  - Numerically ...



- Fighter jet in high-g turn:

Speed: 740 km hr<sup>-1</sup> or 206 m s<sup>-1</sup>.

Radius: 800 m

Centripetal a: 53 m s<sup>-2</sup> or 5.4 g!

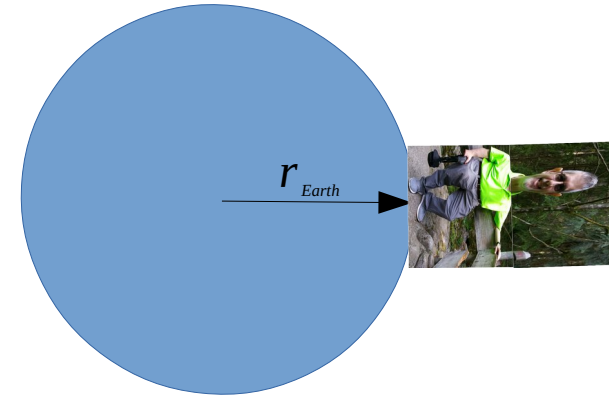


- I'm driving home on Hwy #1:

Speed: 70 km hr<sup>-1</sup> or 19.4 m s<sup>-1</sup>.

Radius: 290 m

Centripetal a: 1.3 m s<sup>-2</sup> or 0.13 g.



- I'm standing on equator on spinning Earth:

Angular Speed: 7.272 X 10<sup>-5</sup> rad s<sup>-1</sup>.

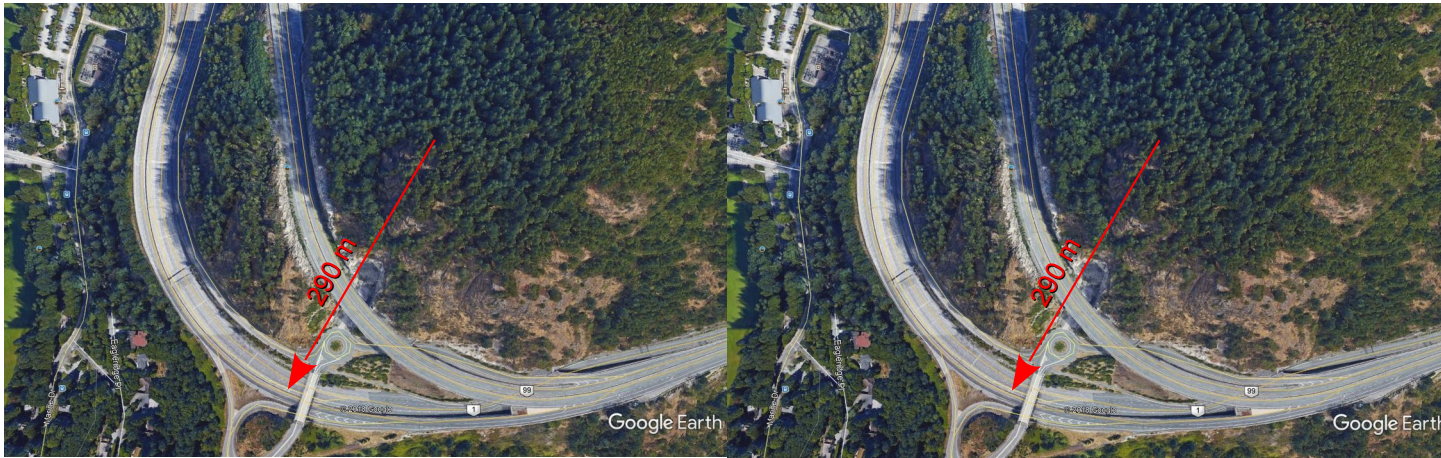
Radius: 6378 km (equatorial)

Centripetal a: 3.4 cm s<sup>-2</sup> or 0.0034 g.

# Velocity & Acceleration

## Centripetal Acceleration of Body on Circular Path:

- Applying centripetal acceleration formula to real situations.
  - Numerically and by noticing its form.



*Notice that only a moderate change in speed results in a pronounced drop in the centripetal acceleration, making driving on that curve much safer in poor conditions (i.e., icy surface) due to dependence on velocity squared.*

- I'm driving home on Hwy #1:

Speed: 70 km hr<sup>-1</sup> or 19.4 m s<sup>-1</sup>.

Radius: 290 m

Centripetal a: 1.3 m s<sup>-2</sup> or 0.13 g.

- I'm driving home carefully on Hwy #1:

Speed: **50 km hr<sup>-1</sup>** or **13.9 m s<sup>-1</sup>**.

Radius: 290 m

Centripetal a: **0.67 m s<sup>-2</sup>** or **0.068 g**.

(Similarly, collisions are less severe by an amount proportional to velocity squared because the *kinetic energy* of the vehicles is also proportional to that.)



# *Fictitious vs True Forces*

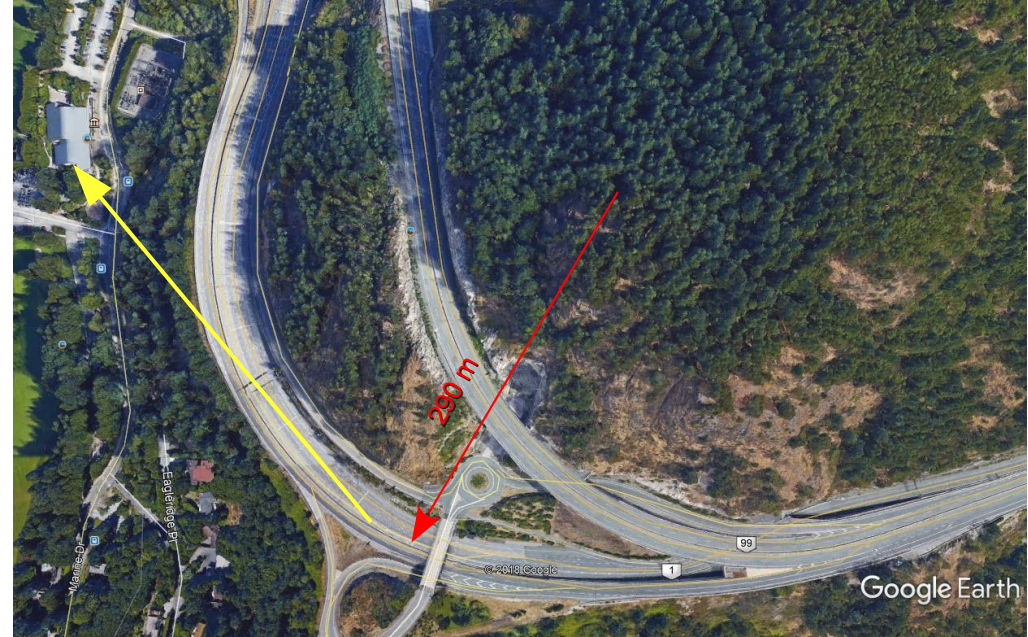
## Centrifugal vs Centripetal:

Test it yourself as car passenger (**NOT** as driver).



While in the curve, confine your vision to the car's interior (i.e., represented by **yellow arrow**):

You'll perceive a "**force**" pushing you toward the outside of the curve, a **centrifugal "force."**



While in the curve, stare at a distant landmark (i.e., represented by **yellow arrow**):

You'll perceive your body's inertia attempting to maintain a straight path but is being **forced** into a circular path by a **centripetal force**.

# *Fictitious vs True Forces*

**Fictitious Forces:** If you and your surroundings (e.g., vehicle) are experiencing acceleration, you're in a non-inertial frame of reference.

The inertia of you and your surroundings will be perceived as “forces,” like the centrifugal and coriolis\* “forces” that are apparent in rotating frames of reference (e.g., a car in a curve or even the surface of the Earth), but are fictional.

\* Resulting in the spinning winds of cyclones and hurricanes.



# *Fictitious vs True Forces*

**True Forces:** In contrast, a person in an inertial frame of reference (i.e., moving at constant velocity) will see the non-inertial frame as accelerating due to an outside true force, e.g., a centripetal force.

# *True Force(s) in a Circular Orbit*

The centripetal and gravitational forces are relevant here.  
BUT, the gravitational force *is* the centripetal force. There is only *one* force here.

A mass  $m$  is in a circular orbit of radius  $r$  around a mass  $M$ , orbiting at velocity,  $v_{orb}$ .

Centripetal Force = Gravitational Force

$$\frac{mv_{orb}^2}{r} = \frac{G M m}{r^2}$$

Note: This expression is not applicable for elliptical orbits because the instantaneous radius of curvature does not necessarily equal the distance between the masses.

# *True Force(s) in a Circular Orbit*

The gravitational force *is* the centripetal force. There is only *one* force here.

After solving for  $v_{orb}$  there is no longer a dependence on  $m$ .

$$v_{orb} = \sqrt{\frac{G M}{r}}$$

Note: This expression is not applicable for elliptical orbits because the instantaneous radius of curvature does not necessarily equal the distance between the masses.



# True Force(s) in a Circular Orbit

**Fun with numbers!**



What is  $v_{orb}$  for low-Earth orbit, say 200 km up?

$$r = (6371 + 200) \text{ km} = 6.571 \times 10^6 \text{ m}.$$

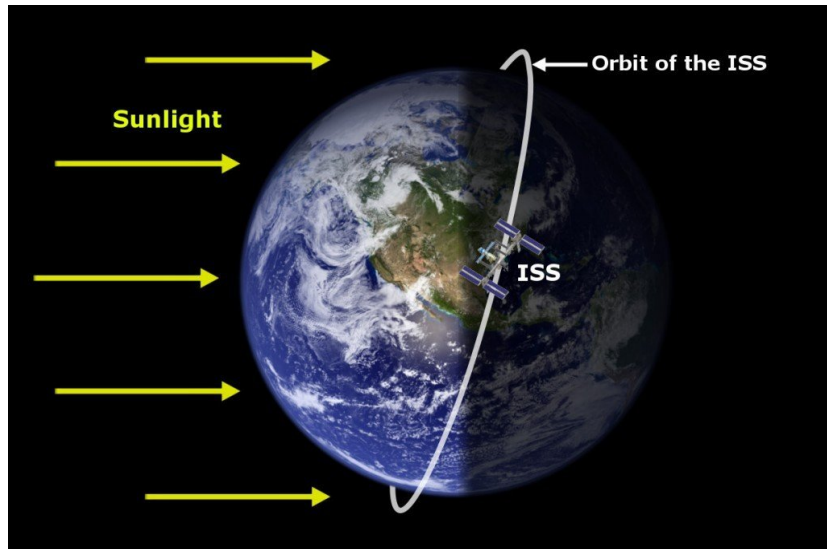
$$M = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$v_{orb} = 7.79 \text{ km/s}$$

$$= 28000 \text{ km/hr}$$

$$= 17400 \text{ mi/hr}$$





# True Force(s) in a Circular Orbit

**Fun with numbers!**



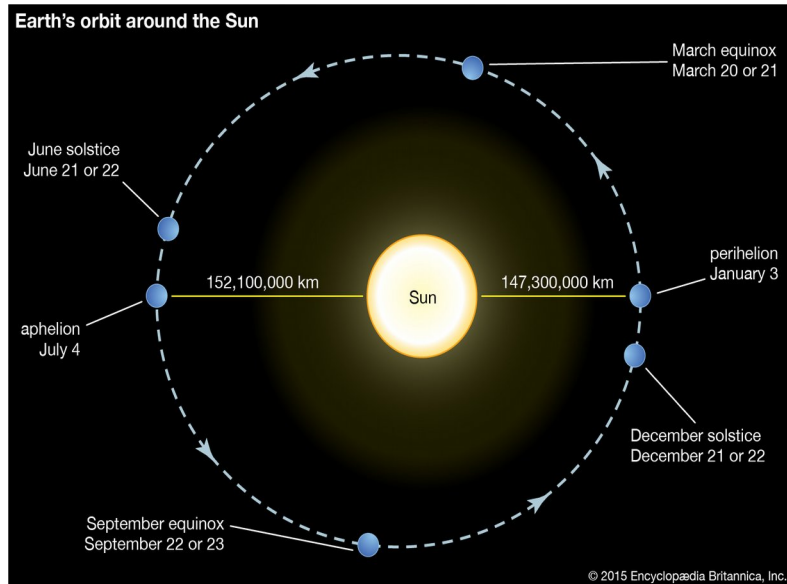
What is  $v_{orb}$  for Earth orbiting the sun?

$$r_{(average)} = 149.6 \text{ million km} = 1.496 \times 10^{11} \text{ m.}$$

$$M = 1.989 \times 10^{30} \text{ kg}$$

$$G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$\begin{aligned} v_{orb} &= 29.8 \text{ km/s} \\ &= 107000 \text{ km/hr} \\ &= 66600 \text{ mi/hr} \end{aligned}$$



NEXT: Escape velocity... but first....

# Work and Energy

**Work:** The product of the force and the displacement it produces.

(If not aligned, then the product of the magnitude of the displacement vector and the projection of the force vector onto that displacement vector.)

E.g., a 60 *kg* person walks up one story of stairs. The force is the weight of the person, or 60 *kg* X 9.8 *m/s*<sup>2</sup>, or 588 *N*. One story is 3 m high. So, the **work** done by climbing that story of stairs is

$$W = mgh = 1764 \text{ Joules.}$$

(If done in 7 seconds, the power output is 1764/7 or 252 *Watts*. This is 252/746 or 0.34 *horsepower*.)



# Work and Energy

**Potential Energy:** This is the **energy** of position within a field, e.g., a mass within a gravitational field (or an electric charge within an electric field).

It is equivalent to the **work** necessary to move a mass (or a charge) to that position within the field.

E.g., that stair-climbing person gained 1764 *Joules* of gravitational **potential energy** (which would be converted to **kinetic energy** if he/she then jumped).



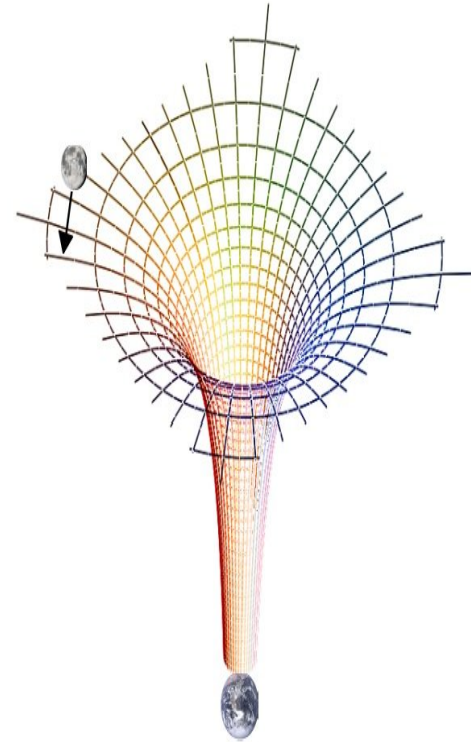
# Work and Energy

**The Potential:** This is the potential energy per unit mass for a gravitational field (in *Joules/kg*).

(For an electric field, it's potential energy per unit charge in *Joules/Coulomb* or *Volts*.)

This is related to the field, which is force per unit mass for a gravitational field (*Newtons/kg* or  $m/s^2$ ).

(For an electric field, it's *Newtons/Coulomb* or *Volts/metre*.)



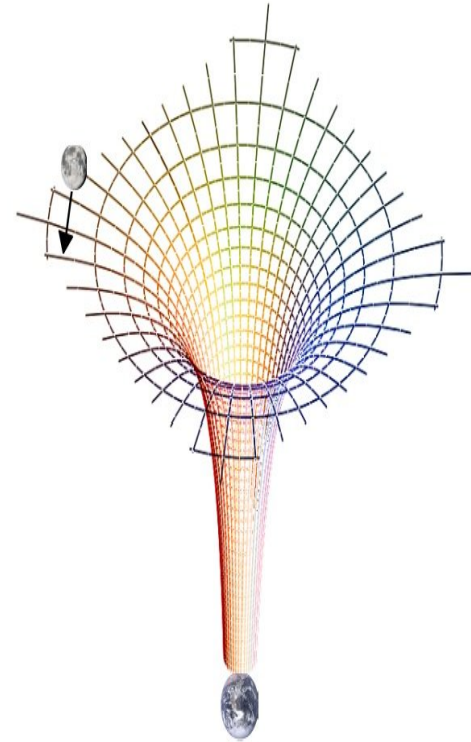
Earth's  
Gravitational  
Potential Well

# Work and Energy

The **field** is the negative gradient (i.e., downhill direction) of **the potential**.

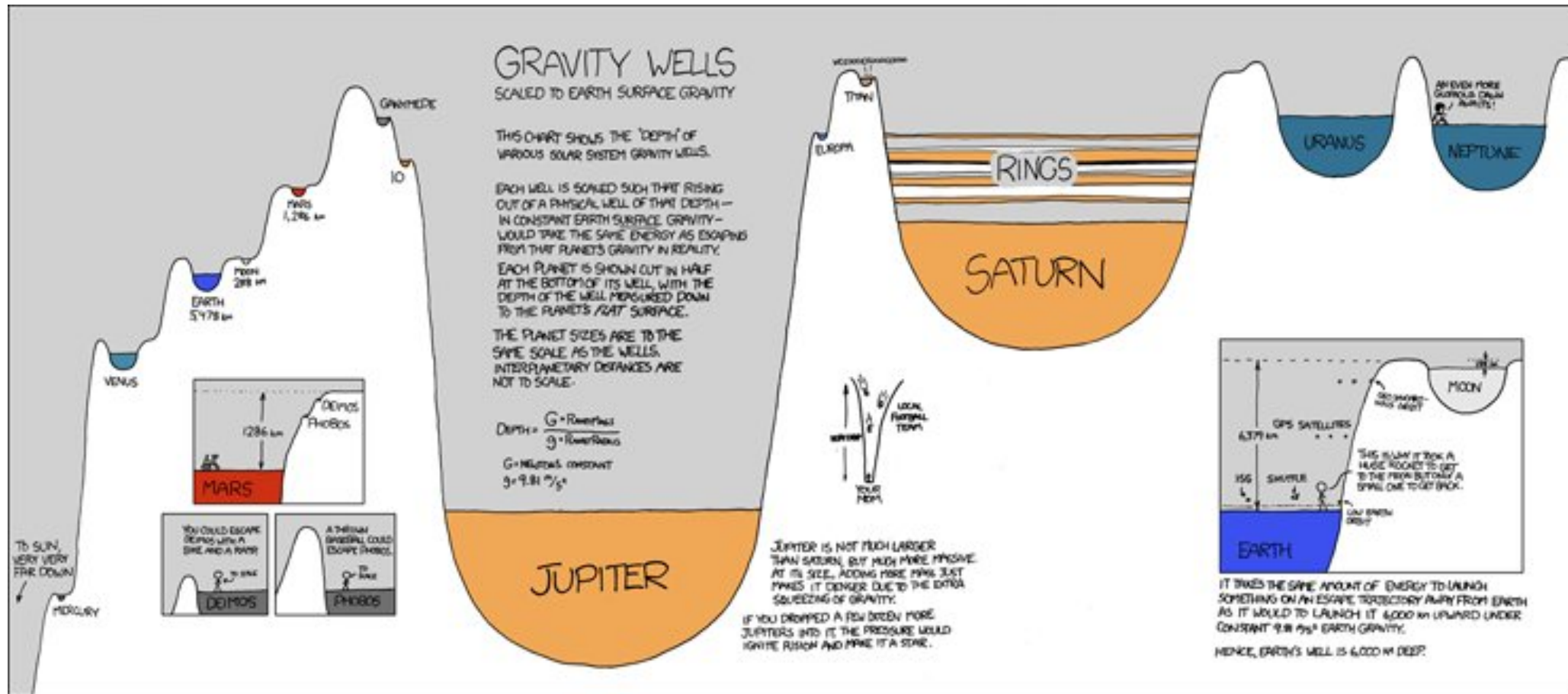
- This is an intuitive way to represent the **field**, because **the potential** represents a surface with hills and valleys, and the **field** would be arrows on that surface that point toward each local downhill direction.

**The potential** is a *scalar* representation of a *vector field*.



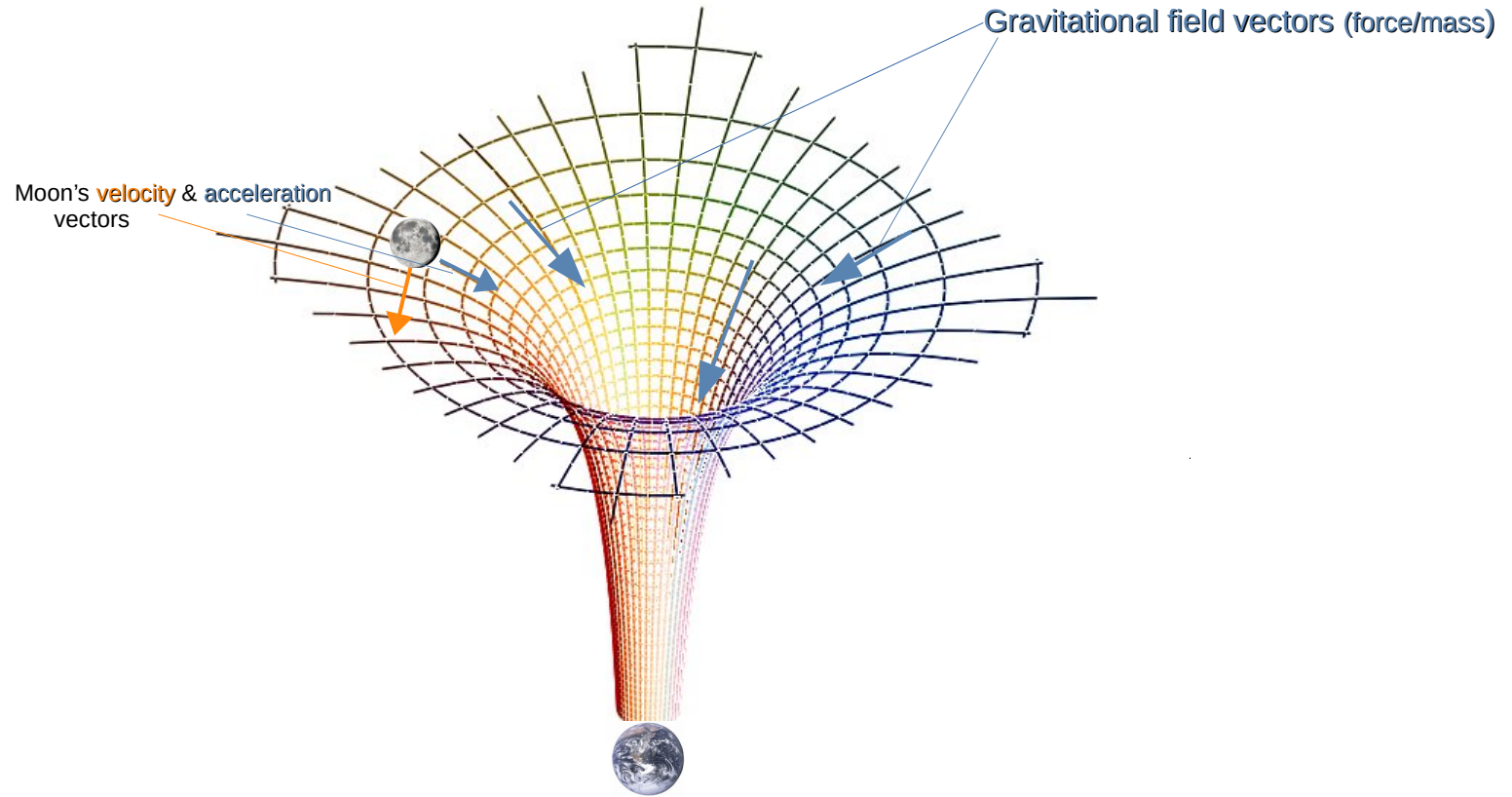
Earth's  
Gravitational  
Potential Well

# Solar System Potential Well(s)



Planet sizes not to same scale as the distances between them.





Earth's Gravitational Potential Well

# Earth's gravitational force:

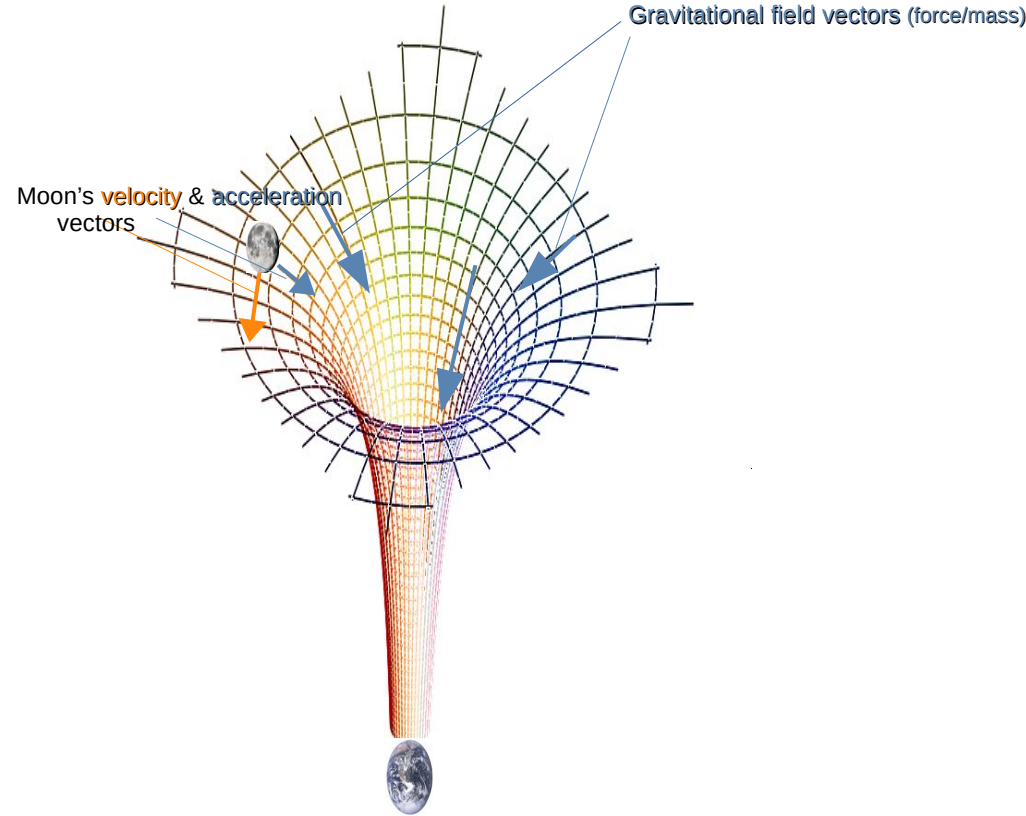
$$\vec{F} = -\frac{G M_{Earth} m}{r^2} \hat{r}$$

The potential energy at distance  $r$  is the work of moving a body from *infinity* to  $r$ . This is equivalent to multiplying the above by  $r$  (in this case).

Or, more accurately, integrating the force from infinity to  $r$ .

The potential energy,  $\Phi$ , is then ...

$$\Phi = -\frac{G M_{Earth} m}{r}$$



Earth's Gravitational Potential Well



Accordingly, **the Potential**,  $\phi$ , is

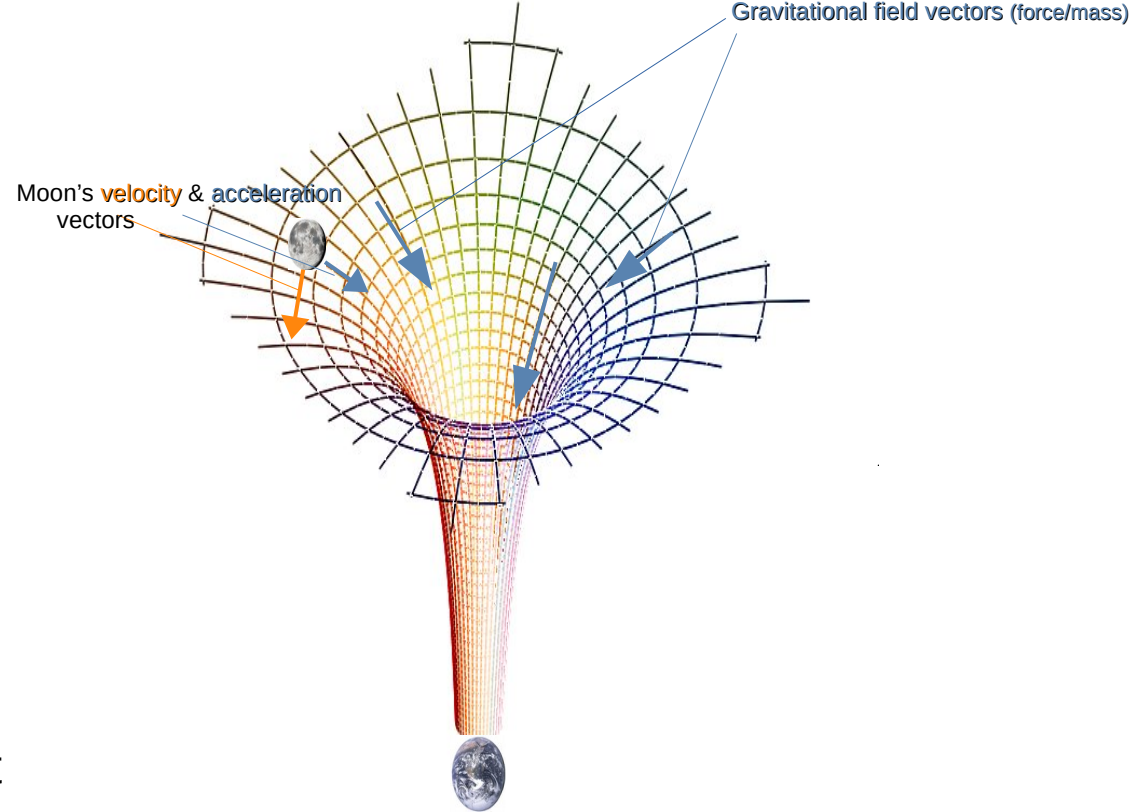
$$\phi = \frac{\Phi}{m}$$

OR

$$\phi = -\frac{G M_{Earth}}{r}$$

**NOTE:** **The potential's** zero-point occurs at  $r = \textit{infinity}$  because the test mass,  $m$ , has *escaped* that potential well.

A bound object has negative **potential**.



Earth's Gravitational Potential Well

NOW! We can have some fun with numbers!!

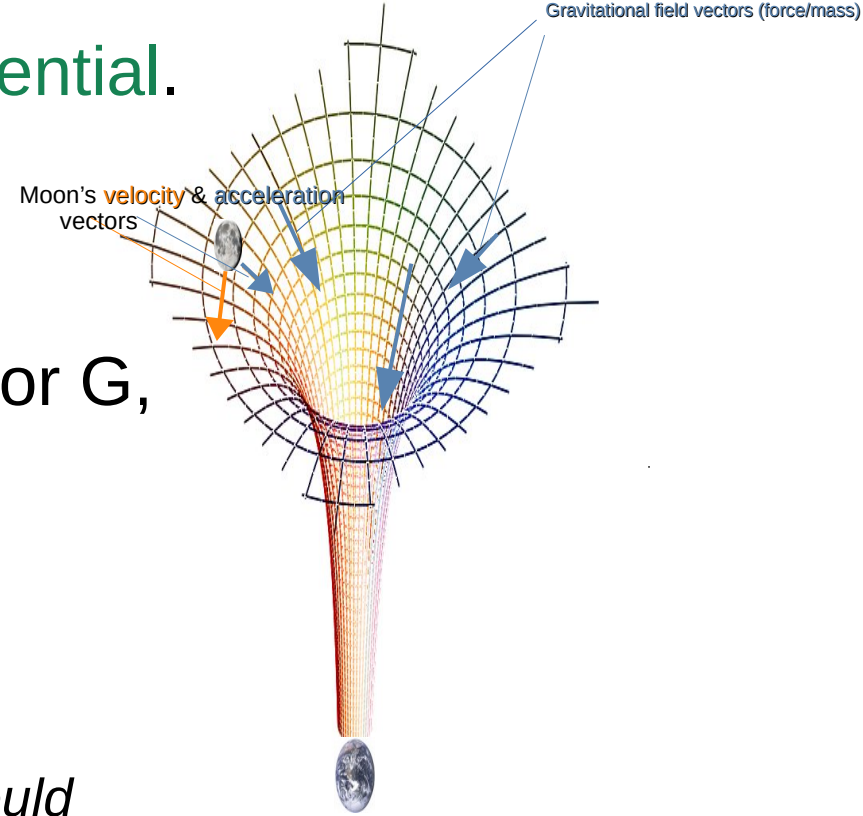
We are all physically bound by that **potential**.  
Exactly how strongly are we bound  
to the Earth's surface?

Using known values for the Earth and for G,  
we find

$$\phi = -62.5 \text{ MegaJoules/kg}$$

So, if we were to add more than 62.5 MJ of **kinetic energy** to each kilogram of your body mass, you could escape the Earth! For a normal-sized adult, this is a few GigaJoules!\* **(Space travel is expensive!)**

\* Equivalent to climbing **2 MILLION STORIES** of stairs!



Earth's Gravitational Potential Well

# Work and Energy

**Kinetic energy** is the **energy** of a body due to its motion.

- What is the **work** required to accelerate mass,  $m$ , to velocity,  $v$  ?
- The force is  $F = m a$  acting over distance  $x = 0.5 a t^2$ . The **work** / **kinetic energy** is then

$$K = \vec{F} \cdot \vec{x} = \frac{1}{2} m a^2 t^2 = \frac{1}{2} m v^2$$



My family on swings, oscillating between peak **potential energy** and peak **kinetic energy**.

2019/05/24

# Work and Energy

**Kinetic energy** is the **energy** of a body due to its motion.

- What is the **work** required to accelerate mass,  $m$ , to velocity,  $v$  ?
- The force is  $F = m a$  acting over distance  $x = 0.5 a t^2$ . The **work** / **kinetic energy** is then

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My family on swings, oscillating between peak **potential energy** and peak **kinetic energy**.

2019/05/24



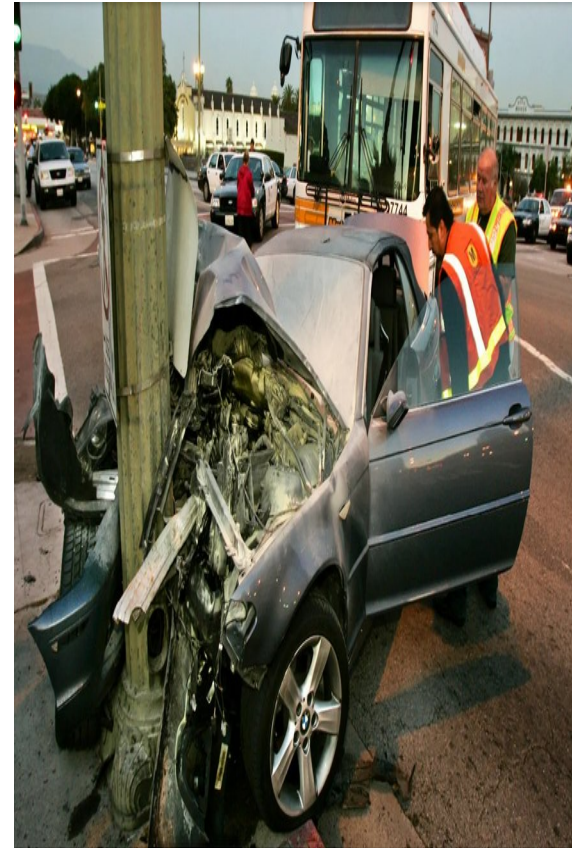
# Work and Energy

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***Notice again that dependence on velocity squared. A moderate change in speed will result in a pronounced change in kinetic energy and, therefore, in the damage in collisions! Drive safely/slowly!***



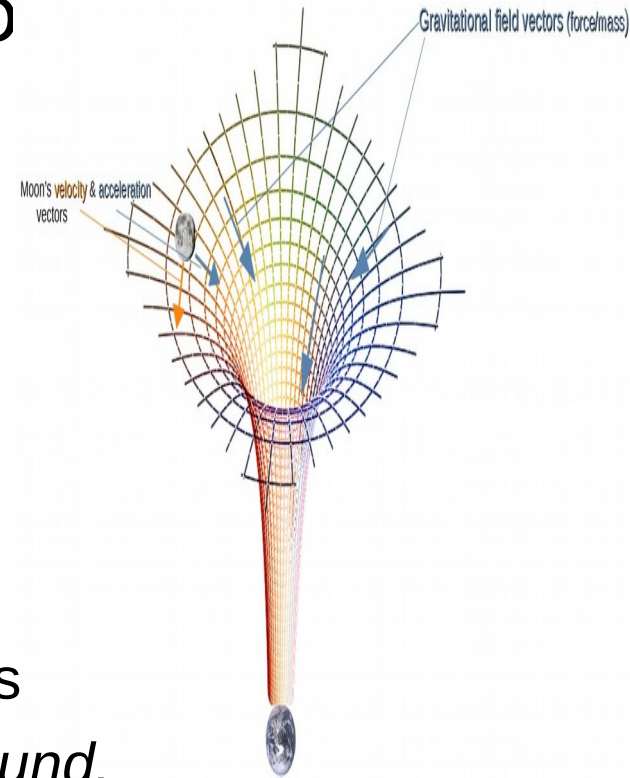
Too much **kinetic energy**.

# Total Energy (Kinetic + Potential)

Adding the **kinetic energy** of mass  $m$  to its **potential energy** yields the **total energy**,  $E_{TOT}$ , of mass  $m$  within the gravitational field of mass  $M$ :

$$E_{TOT} = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

- Whether  $E_{TOT}$  is negative, zero, or positive determines whether an orbit is *bound*, *marginally bound*, or *unbound*.
- This also determines the orbital trajectory (*elliptical*, *parabolic*, *hyperbolic*).



Earth's Gravitational Potential Well

# Total Energy (*Kinetic* + *Potential*)

When  $E_{TOT} = 0$ , the velocity,  $v$ , is the escape velocity,  $v_{esc}$ , the minimum velocity necessary for  $m$  to marginally escape the gravitational field of  $M$ :

$$0 = \frac{1}{2} m v_{esc}^2 - \frac{GMm}{r}$$

Solving for  $v_{esc}$  yields...

$$v_{esc} = \sqrt{2 \frac{GM}{r}}$$

**DÉJÀ VU!!**

Looks like the expression for  $v_{orb}$  !!

In fact, we find...

$$v_{esc} = \sqrt{2} v_{orb}$$



Therefore, if you're in orbit, then you only need to increase your velocity by **41%** (a factor of 1.41) to escape the gravity of mass  $M$  (regardless of  $r$ )!

In other words, you already have **half** the **kinetic energy** needed to escape.



# Total Energy (Kinetic + Potential)

## Fun with numbers!



What is  $v_{esc}$  for escaping from Earth's surface?

$$v_{esc} = 2^{0.5} \times 7.79 \text{ km/s} = \mathbf{11.0 \text{ km/s}}$$

$$= \mathbf{39660 \text{ km/hr}}$$

$$= \mathbf{24600 \text{ mi/hr}}$$

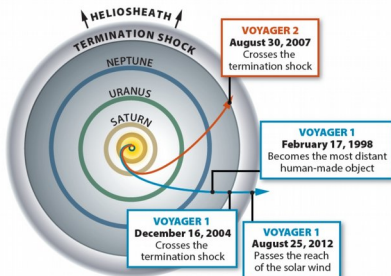


What is  $v_{esc}$  for escaping from the solar system starting at Earth?

$$v_{esc} = 2^{0.5} \times 29.8 \text{ km/s} = \mathbf{42.1 \text{ km/s}}$$

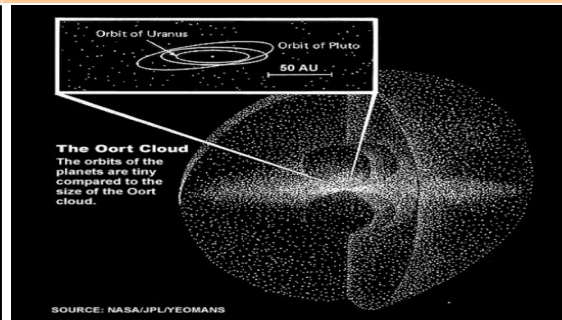
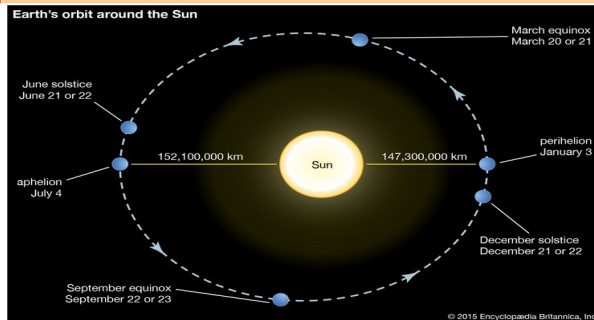
$$= \mathbf{152000 \text{ km/hr}}$$

$$= \mathbf{94300 \text{ mi/hr}}$$



# Total Energy (*Kinetic* + *Potential*)

	$E_{\text{TOT}} < 0$	$E_{\text{TOT}} = 0$	$E_{\text{TOT}} > 0$
Orbit Type	Bound	Marginally bound	<b>Unbound</b>
Trajectory	Elliptical	Parabolic	Hyperbolic
Eccentricity	$0 \leq e < 1$	$e = 1$	$e > 1$
Examples	Earth orbiting sun ( $e=0.0167086$ )  Short-period comets	Long- "Period" Comets	1I/ 'Oumuamua ( $e = 1.19951 \pm 0.00088$ )  2I/ Borisov ( $e = 3.3575 \pm 0.0004$ )



# *I. Basic Physics:*

- Newton's laws of motion
- Newtonian gravity
- Weight vs Mass
- Velocity & Acceleration
  - Linear acceleration
  - Centripetal acceleration
- Fictitious vs True Forces
- Velocity in a circular orbit
- Work and Energy
  - Potential Energy
  - Potential
  - Kinetic Energy
- Total Energy
  - Escape velocity
  - Orbital trajectories

## *II. Orbital Basics and Real Orbits:*

# NEWTON'S CANNON



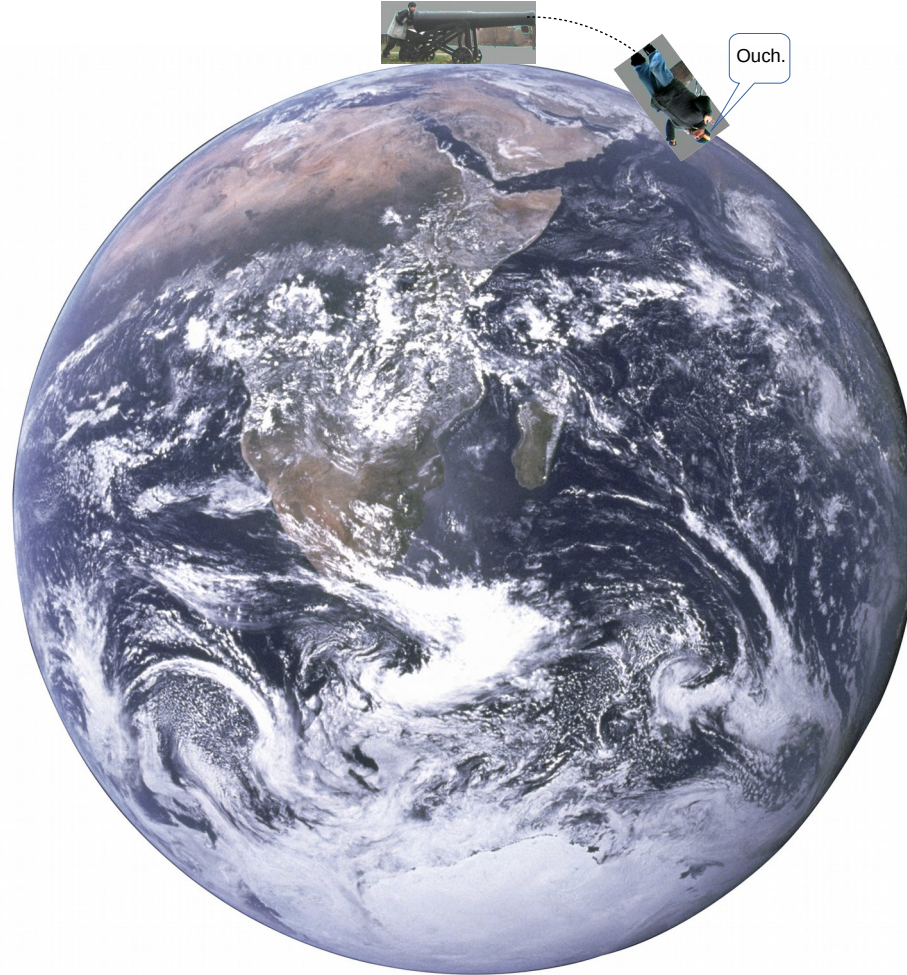
Dad and Dylan at Fort George, Niagara-on-the-Lake, Ontario, Canada, Jan /2006, Photo: Narissa Wall



# NEWTON'S CANNON



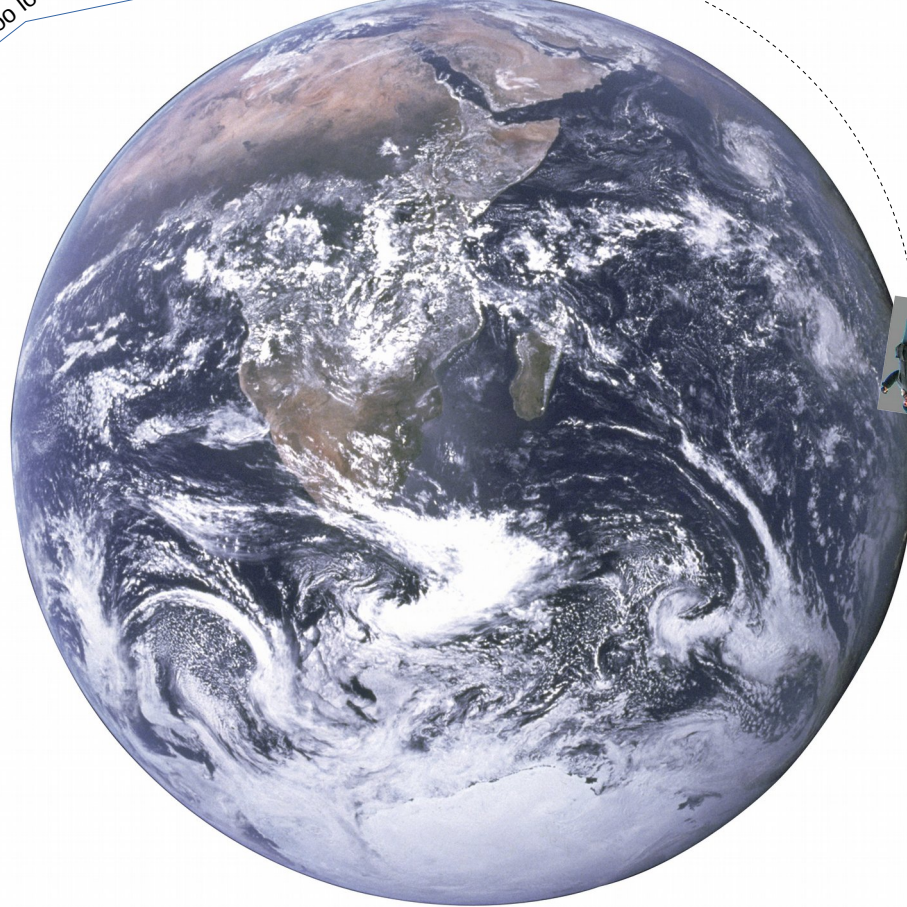
# NEWTON'S CANNON





# NEWTON'S CANNON

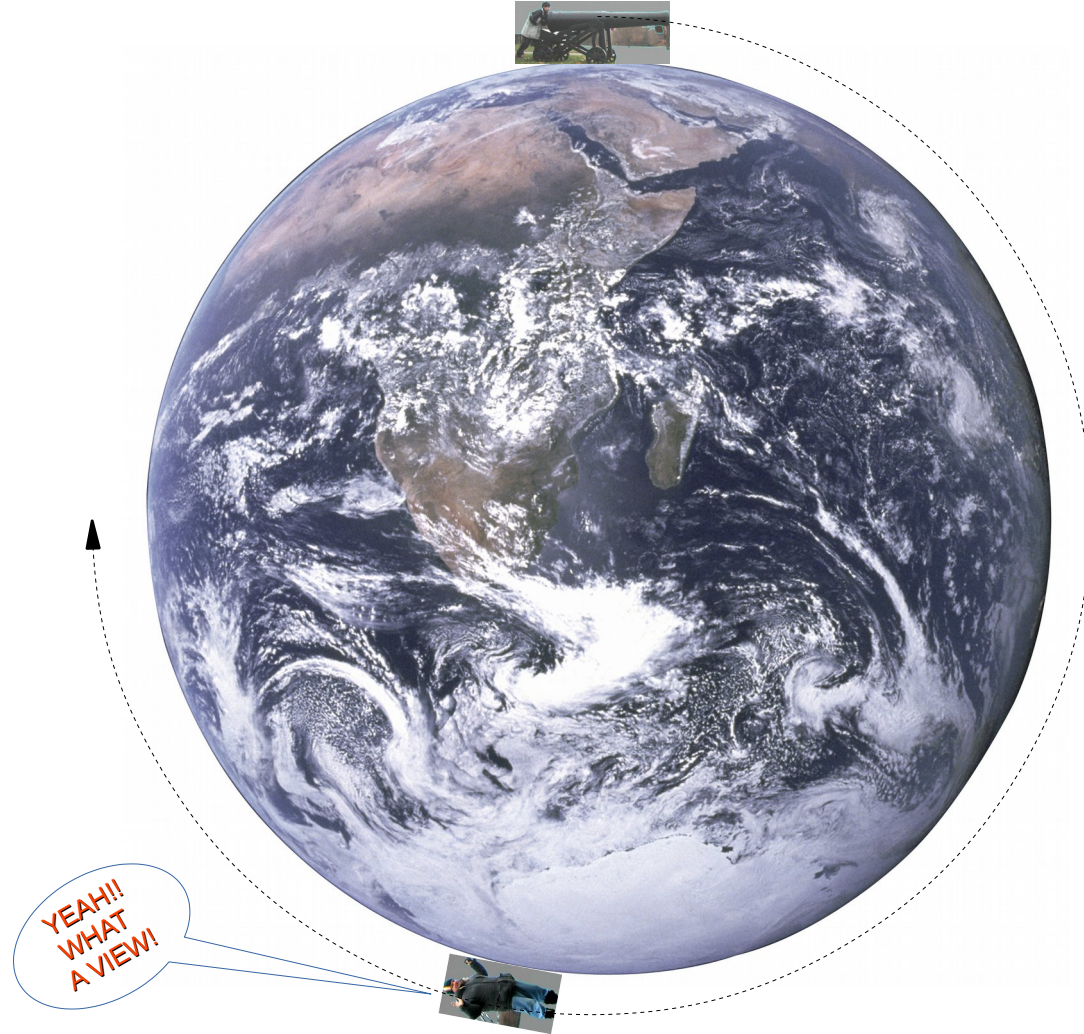
Sorry, Dad. Muzzle velocity  
is still too low.



Ouch!



# NEWTON'S CANNON



# *Kepler's Laws of Planetary Motion*

- 1) The orbit of a planet is an ellipse with the sun at one of the two foci.
- 2) A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. (*Conservation of angular momentum.*)
- 3) The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.  $T^2 \propto a^3$

- Kepler's Laws of Orbital Motion – Newton's law of gravitational motion.
  - Newton's Cannon – free fall and weightlessness.
- Orbits and their elements – elliptical, parabolic, hyperbolic orbits
  - Periasis, apasis points
  - Discussion of eccentricity
  - Use blender 3D model to demonstrate.
- Complications in real orbits
  - non-negligible masses – orbit common centre of mass
  - non-spherical and extended
  - more than 2 bodies
  - Poynting-Robertson effect
  - Yarkovsky effect

- Examples of real orbits -
  - artificial satellites
  - moon around earth
  - earth around sun
  - other planets around sun
  - exoplanets
- Additional considerations of orbits (might be combined with above)-
  - Tides
  - Lagrangian points
  - Seasons
  - Observations of solar system from Earth – retrograde loops
  - Precession
  - Eclipses
  - Occultations
  - Milankovitch cycles

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